PLATE ANALOGY OF BEAM GRILLAGE FOR DYNAMIC ANALYSIS

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By

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CERTIFICATE

Certified that this work on "PLATE ANALOGY OF BEAM GRILLAGE FOR DYNAMIC ANALYSIS" by Rajeshwar Dayal Sharma has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

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22.3.73

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RAJESHWAR DAYAL SHARMA

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NOMENCLATURE

a	Length of the beams in x direction.
a_1	Spacing of beams in x direction.
b	Length of beams lying in y direction.
b_1	Spacing of the beams lying in y direction.
c	Fineness parameter.
c_1	Torsional rigidities of beams in x direction.
c_2	Torsional rigidities of beams in r direction.
e_r, g_r	Constants appearing in equation (3.33)
f_{mn}	Parameter defined in 3.61
h	Thickness of the equivalent plate.
m_b	Mass per unit length of a beam of an isotropic grid.
m_x, m_y	Mass per unit length of beams lying in x and y-directions of an orthotropic grid.
p, q	Nondimensional flexural rigidity and thickness parameter for square isotropic grid.
p_N, q_N	Nondimensional tension and thickness parameter of the cable net-work.
p_{mn}	Natural frequency (rad/sec) of vibration.
q_{xy}	Load per unit area.
γ_b	Weight density of the material of the beam constituting the isotropic grid.

γ_x, γ_y Weight densities of beams, lying in x and y directions, of an orthotropic grid.

ν Poission ratio for the material of the isotropic plate.

$\nu_x \; \nu_y$ Poission ratios for the material of an orthotropic plate with its edges along x and y axis.

ρ_p Mass density of the equivalent plate.

ρ_b Mass density of the material of the beam.

A_b Area of cross section of the beams constituting the isotropic grid.

A_p Area of the surface of the plate.

B_1 Flexural rigidities of beams lying in x direction of the grid.

B_2 Flexural rigidities of beams lying in y direction of the grid.

B_{mn} Amplitude of vibration in m,n mode.

D Flexural rigidity parameter of the equivalent plate.

D_i Flexural rigidity parameter of the isotropic plate.

D_o Flexural rigidity parameter of the orthotropic plate, S.S. on the edges, defined in eq. 3.9.

D_{oF} Flexural rigidity parameter of the orthotropic plate, clamped on the edges, defined in eq. 3.36.

D_x, D_y, D_1, D_{xy} Flexural rigidity constants for orthotropic plate defined in 3.8.

E_p	Elastic constant for the material of isotropic plate.
E_x, E_y	Elastic constants for the material of beams, lying in x and y direction, in an orthotropic grid.
G	Shear modulus of the material of the equivalent plate.
G_r	Constant defined in eq. 3.44.
$G1_r$	Constant defined in eq. 3.45.
$H_m^{N_y}$	Constant defined in eq. 3.42.
$H_n^{N_x}$	Constant defined in eq. 3.43.
I_b	Moment of inertia of the beam constituting isotropic grid.
I_x, I_y	Moments of inertia of the beams lying in x and y direction in an orthotropic grid.
Jx, Jy	Polar moments of inertia of the beams lying in x and y direction in an orthotropic grid.
K_{xi}	Kinetic energy of ith beam lying in x direction.
K_{xb}, K_{yb}	Kinetic energies of beams lying in x and y directions respectively.
K_g	Kinetic energy of the grid.
K_p	Kinetic energy of the equivalent plate.

M_i	Bending moment at joint i.
S_i	Shear force at joint i.
U_{xi}	Strain energy of ith beam lying in x-direction.
U_{xb}, U_{yb}	Strain energies of beams lying in x and y-directions respectively.
U_g	Kinetic energy of the grid.
U_p	Strain energy of the equivalent plate.
$X_r(x)$	Beam function defined in eq. 3.33.

ABSTRACT

PLATE ANALOGY OF BEAM GRILLAGE FOR DYNAMIC ANALYSIS

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In the present work dynamic analysis of beam grillage has been undertaken. The analysis is based on the rational proposition of orthotropic plate analogy formulated on the basis of energy equivalence, replacing the Timoshenko's plate analogy which is an intutive proposition.

The analogy is validated for the case of orthotropic grid with simply supported edges by comparing the results of the frequencies with those obtained by Finite Element method. An attempt has been made to extend the proposition to the case of grillage with clamped edges but is limited to fundamental frequency of the orthotropic grillage.

CHAPTER I

INTRODUCTION

The use of computer for dynamic analysis of beam grillage is very common now a days. This is time consuming and costly. In the present work, an attempt has been made to establish plate analogy of beam grillage such that the analysis is simplified.

A grillage is a structure consisting of two sets of orthogonal or nonorthogonal beams rigidly connected at the intersections. Some of its applications are in building floors (concrete slabs embedded in grillage), ship decks, bridge floors, missile ground facilities, electronically steerable radar systems etc. Several studies have been made of static and dynamic analysis of beam grillage. These are classified into two groups.

(a) Analogy methods

(b) Methods using discrete approach

Discrete approach is very suitable for analysis of such systems but is laborious and time consuming. Furthermore if one uses consistent mass matrix approach, it becomes difficult to solve higher order grids due to limited memory space of the computer. Terms like isotropic and orthotropic grillages are commonly used in this dissertation and are explained as follows:

Isotropic beam grillage:

A grillage made of two sets of orthogonal beams, equally and uniformly spaced in both directions and having same geometric and material properties, is called an isotropic beam grillage.

Orthotropic beam grillage:

An orthotropic grillage is made of two sets of orthogonal beams, each having different geometric or material properties, with uniform spacing of the beams in each direction but the spacing in one direction may be different from that in the other.

The above terms have been defined in a manner that while proposing the analogy, these have similar meanings as those used for plates.

1.1 STATEMENT OF THE PROBLEM:

The present work deals with formulating an analogy for the dynamic analysis of the beam grillage for finding its material frequencies. As mentioned previously discrete analysis is time consuming and as such an attempt is to find an analogy of the physical system (grid). Thus the problem under consideration is concerned with finding a physical continuous system equivalent to that of beam grillage in m, n mode. This implies finding unknown parameters, the flexural rigidity and the thickness, of

the equivalent plate in terms of the known parameters of the grillage.

The statement of the problem being undertaken in this work is thus

"To establish the plate analogy of the beam grillage for its dynamic analysis".

1.2 PROPOSED ANALOGY:

The analysis developed is based on formulation of energy equivalence. Thus for dynamic analysis of a grillage, it is said to be replaced by an equivalent continuous system, plate, if the maximum strain and kinetic energies of the two systems are equal. In the present work, transverse vibration of the grillage is being considered, as such, equating the maximum strain energy of the grillage to that of the plate in bending and equating maximum kinetic energy of the grillage to that of the plate, yield the required unknown parameters of the equivalent plate in terms of the known parameters of the grillage.

The parameters thus found, are substituted in the frequency equation of the equivalent plate which directly gives the frequencies of the given beam grillage.

In the present work, beam grillages with the following two sets of boundary conditions are considered.

1. Grillage, simply supported on the boundaries and
2. Grillage clamped on the boundaries.

However the proposition is general and can be applied to grillage having any set of boundary conditions if the corresponding equivalent plate solution is known.

In the second chapter the review of the earlier work is given. Third chapter deals mainly with the formulation and method of solution of the problem. Case studies are presented in chapter four and the results obtained from discrete analysis and analogy method are compared. These results are also compared with the results of earlier work done. Chapter V concludes with the validity of the analogy proposed.

CHAPTER II

LITERATURE REVIEW

It has already been mentioned that the present work deals with the dynamic analysis of grillage. Several studies have been made on the subject and these can be divided in two groups:

1. Studies using discrete approach and
2. Studies which replace the grillage by an equivalent continuum, plate.

In this chapter, a review of the earlier work on grillages is given.

2.1 DISCRETE APPROACH:

In these methods, the system under consideration is divided into small elements such that the overall behavior of the system is unchanged. The stiffness and mass properties of these elements are determined which after being assembled for the whole system give its mass and stiffness properties.

The earlier work on the vibration of grillages is known to be that of Cox and Denke[5] who formulated the problem into matrix form, suitable for use with a digital computer.

J.P. Ellington & H. McCallion[6] have taken into account the discrete nature of the beams and considered the grillage system to be made up of two orthogonal sets of beams with equal spacing attached at the joints so that no moments, bending or torsional, are transmitted from one set to another. The mass of the beams is taken as lumped at the joints (thereby reducing the problem to finite degree of freedom) so that there is an equal point load at each internal joint. Taking the bending moments and deflections under each load as unknowns, the equations of continuity and equilibrium, taken with appropriate boundary conditions, furnish the required frequency equation. They used the finite difference technique, coupled with three moment equations, for developing frequency equation.

A more general solution was presented by Thein Wah[23], who extended the method of J.P. Ellington and H. McCallion[6] by taking distributed mass of the beams instead of lumping the masses at the joints. He further extended the method so as to investigate the effect of St. Venent torsional inertia on the transverse vibrations. However due to complexity of the formulation, the solutions obtained are for bending deformations only.

It is to be remembered that the solution of Ellington and McCallion[6] is not an approximate solution but rather an exact solution of an approximate formulation. Possibly as a consequence of this, the lowest order

frequencies are sometimes under estimated and sometimes over estimated.

Thein Wah's[23] work was extended by Chang and Michelson[3] and the complicated equations were simplified with the application of Raleigh Ritz method. Chang's[3] method is specially useful for the vibration analysis of the ship structures.

Miller and Kamm[16] employed the new technique of matrix formulation for the dynamic response of grid structures by using the lumped parameter method for which the static spring is composed of flexural and torsional stiffness coefficients formulated from the classical slope deflection equation.

Cheng[4] analysed the torsional-flexural vibrations by using the general dynamic matrix of distributed mass system for the orthogonal or nonorthogonal grillages composed of either prismatic bars or thin walled open section members. The differential equations are derived and the dynamic stiffness coefficients are then formulated for which a general computer programm is developed that is applicable to various grid works with different boundary conditions of fixed, free, or hinged support. The effect of rotary inertia, shear and bending deformation and that of St. Venent torsional inertia on the natural frequencies are taken into account for the grid work of prismatic bars.

For the grillage with wide flanged members, the additional effect of warping and the associated effect of longitudinal and transverse-shear deformation of the flange on the vibration are also included. The effect of the sectional dimensions on frequencies is also considered in the method of Cheng[4] which consequently leads to accurate solutions of higher modes.

It should be remembered that the method of Cheng[4] presents difficulties when the order of the grid becomes high due to limited computer memory space.

2.2 STUDIES WHICH REPLACE THE GRILLAGE BY AN EQUIVALENT PLATE:

For beam grillages, the idea of replacing a discrete system with an equivalent continuum has received an increasing attention. In these studies, the actual beam grid is replaced by an equivalent plate which in turn gives the solution of the grillage.

Several studies have been made in which the grid is replaced by an equivalent orthotropic plate. For static analysis Timoshenko[24] considered the grid, shown in Figure 2.1 and assumed that if the distances a_1 and b_1 between the beams are small in comparison with the dimensions a and b of the grid and if the flexural rigidity of each of the beams parallel to the x axis is equal to B_1 and that of the beams parallel to y axis is equal to B_2 , the plate

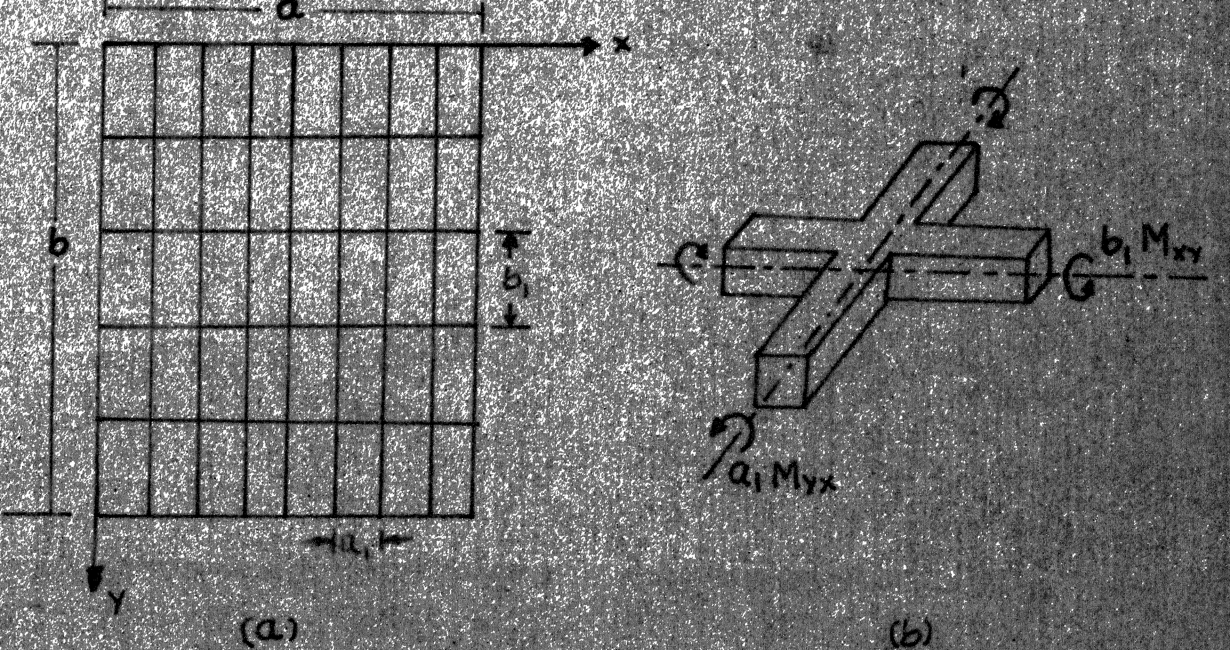


Fig. 2-1. Grid of [24]

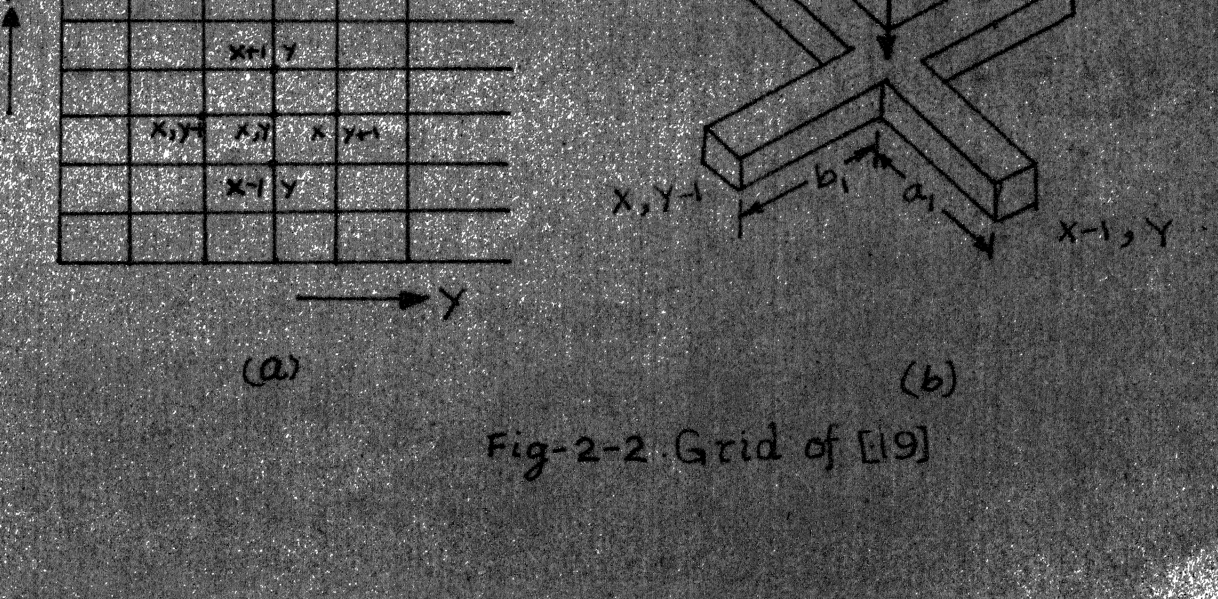


Fig-2-2. Grid of [19]

parameters are written as

$$D_x = B_1/b_1 \qquad D_y = B_2/a_1 \qquad (2.1)$$

He expressed the quantity D_{xy} in terms of the torsional rigidities c_1 and c_2 of the beams parallel to x and y axes respectively as follows:

$$M_{xy} = \frac{c_1}{b_1}\frac{\partial^2 w}{\partial x \partial y} \qquad M_{yx} = -\frac{c_2}{a_1}\frac{\partial^2 w}{\partial x \partial y} \qquad (2.2)$$

Since the beam is considered as a one dimensional continuum, poisson's ratio ν is zero and the quantity D_1 for the grillage, defined below,

$$D_1 = \frac{E_x \nu_y h^3}{12(1 - \nu_x \nu_y)}$$

is also zero.

Thus the parameters of the equivalent orthotropic plate are known in terms of the known parameters of the given grid and the static analysis of the grid can be done easily.

Using Timoshenko's results, Taraporewalla[22] found static deflection of the grillages by considering the plate analogy. The relations given by Timoshenko[24] were purely intutive and gave no indication of the accuracy of the method. Further he assumed that the spacing of the beams in the grid is very small, which may not be valid in most of the grillages.

Renton J.D.[19] gave the theoretical basis for the relations given by Timoshenko. He took advantage of the resulting regularity of the grids and the fact that a large number of beams are used in their formation. He used the combination of "finite difference", and differential calculus, the two being related by Taylor's expansion. This technique gives differential equation for the deflection of an equivalent continuum.

In case of planer grids, the resulting equations are biharmonic and are analogous to those of plates, so that known solutions of the plate problems can be used to solve grid work systems.

For the rectangular mesh grid, shown in Figure 2.2, he obtained the following equation

$$\frac{E_x I_x}{b_1}\frac{\partial^4 w}{\partial x^4} + \left(\frac{GJ_x}{b_1} + \frac{GJ_y}{a_1}\right)\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{EI_y}{a_1}\frac{\partial^4 w}{\partial y^4} = q_{xy} \quad (2.3)$$

in which $E_x I_x$ and GJ_x are the flexural and torsional rigidities of beams of span a_1 in the x-direction and $E_y I_y$ and GJ_y are flexural and torsional rigidities of beams of span b_1 in y-direction, q_{xy} is load per unit area. In all the cases, as the mesh sizes a_1 , b_1 tend to zero, solutions tend to the corresponding plate solutions.

Renton[19] made the same assumption as that of Timoshenko that is the spacing of the beams in the grid is small in comparision to overall dimension of the grid and

while deriving equation (2.3) he ignored terms proportional to the square of the ratio of the mesh size to the overall dimensions of the grid.

In his further work, Renton[20] gave 1st. and 2nd. order continuum approximation for various forms of space grids. The introduction of 2nd. order terms not only gives more accurate results but also indicates the range of applicability of the concept of analogous continua. The method is inappropriate for grids when the fineness parameter $c = a/a_1$, where a is the side of boundary and a_1 is the length of unit step, is less than four because the convergence is not assured.

William R. Flower and Lewis C. Schmidt[26] analysed a particular form of double layer space truss by replacing it with an equivalent plate. From consideration of the partial differential equation derived from the equilibrium equations of the truss by means of Taylor's expansion, it is possible to express the elastic constants for the equivalent plate in terms of the grid parameters. This is possible if the derivatives of the deflections are bounded which implies that the Taylor series expansion is valid.

In the paper of Cheng[4], the results for the frequency parameters for a 3 x 3 square uniform orthogonal grid are given, which have been obtained from orthotropic plate[7] solution using plate analogy. The results compare well with the result obtained from various discrete methods.

Note: Details of paper 7 are not available.

CHAPTER III

MATHEMATICAL FORMULATION AND METHOD OF SOLUTION

As mentioned, the analogy is based on the proposition that plate with boundaries same as that of grid is dynamically equivalent to grid if its maximum kinetic and potential energies are same as that of grid. This equivalence gives two required unknown parameters, flexural rigidity, D and the thickness h of the equivalent plate, in terms of known parameters, elastic constant E, moment of the area I and given dimensions of the grillage. Once the parameters of the equivalent plate are known, well established solution of the plate can be used to find the natural frequencies of the grid.

3.1 MATHEMATICAL FORMULATION:

In the present work, analogy for the class of grillages is considered for which the corresponding equivalent plate solutions are known. Since exact solution is kmown for the plate with simply supported edges, the grillages with simply supported ends are first considered. Furthermore approximate solutions of the plate clamped on the edges is also known, as such having established the equivalence for grillage with simply supported edges, the analysis has been extended to grillages clamped on the edges.

3.1.1 Assumptions:

The grillage is analysed for transverse vibrations wherein any element is considered to be in bending. The analysis is based on certain assumptions, which are summarized below:

1. The grillage is torsionless.

2. The rotatory inertia effects and the shear effects in the transverse vibrations of the grillage are neglected.

3. The transverse deflections of the grillage are considered to be small.

4. The equivalent plate follows Kirchoff's hypothesis which implies.

   (a) A normal which is perpendicular to the middle plane remains normal to the deformed middle surface and does not change its length.

   (b) Middle plane of the plate is not strained.

   (c) Normal stress transverse to the middle plane can be neglected.

The analogy proposed implies that an orthotropic rectangular grid is replaced by an equivalent orthotropic plate as such an isotropic rectangular grid is replaced

by an equivalent isotropic plate. In particular, a square isotropic grid is to be considered by its equivalent isotropic square plate.

Consider first the grillage with simply supported ends.

3.1.2 Grillage simply supported on the boundaries:

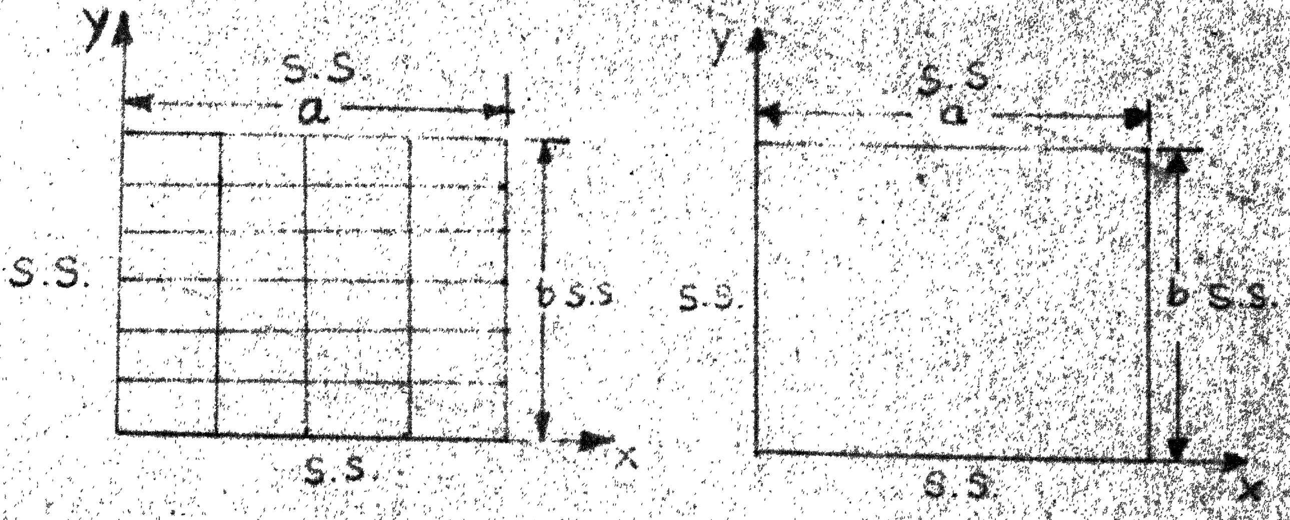
Consider an orthotropic beam grid work shown in figure 3.1a, of length a and breadth b with number of beams N_x and N_y and with flexural rigidities E_xI_x and E_yI_y, in x and y direction respectively. According to the proposed plate analogy, the grid can be replaced by an equivalent orthotropic plate with same set of boundary conditions, refer figure 3.1-b. To obtain energy expressions for the vibrating elastic system, its model functions are required. To start with, it is assumed that the model functions of the grid are same as those of the equivalent plate. It is well known that $(m,n)^{th}$ mode eigen-functions of the plate, simply supported on the edges, is given by

$$w(x,y,t) = B_{mn} \operatorname{Sin} \frac{m\pi x}{a} \operatorname{Sin} \frac{n\pi y}{b} \operatorname{Sin} p_{mn}t \qquad (3.1)$$

where B_{mn} is the amplitude and

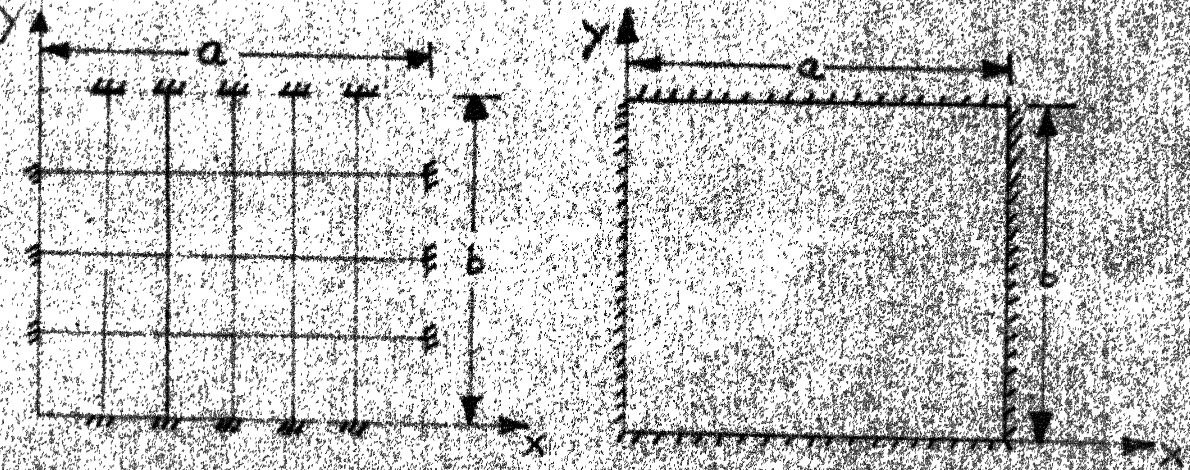
p_{mn} is natural frequency in m, n mode.

It can be established that mode shapes of the grid found by finite element method fit in closely with modal functions of the plate at all the discrete points. As such



(a) (b)

Fig. 3-1



(a) (b)

Fig.3-2. Grid and Equivalent Plate

the mode shape of a beam of the grid can be adopted as that obtained from the modal function of the plate correspondir to its discrete value of x or y. One can thus write the expression for the mode shape of ith beam lying along x-direction as

$$w_i\left(x, \frac{bi}{N_x+1}, t\right) = B_{mn} \operatorname{Sin} \frac{m\pi x}{a} \operatorname{Sin} \frac{n\pi}{b}\left(\frac{bi}{N_x+1}\right) \operatorname{Sin} p_{mn}t \qquad \ldots\ldots \quad (3.2a)$$

where , $i = 1, 2, \ldots\ldots\ldots, N_x$

or

$$w_i\,(x,t) = B_{mn} \operatorname{Sin} \frac{m\pi x}{a} \operatorname{Sin} \frac{n\pi i}{N_x+1} \operatorname{Sin} p_{mn}t \qquad (3.2b)$$

Similarly, the mode shape for the jth beam lying along y-direction can be written as:

$$w_j\,(y,t) = B_{mn} \operatorname{Sin} \frac{m\pi j}{N_x+1} \operatorname{Sin} \frac{n\pi y}{b} \operatorname{Sin} p_{mn}t \qquad (3.3)$$

where $j = 1, 2, \ldots\ldots\ldots\ldots, N_y$

The grillages, simply supported on the edges, can be classified as orthotropic and isotropic and are discussed separately.

A- Orthotropic grid:

An orthotropic grid, as defined earlier, is made of two sets of orthogonal beams, each set differing from the other in its material or geometric properties. Whereas an orthotropic material has two preferred directions, the material properties remain the same if the

directions are reversed. For such materials, the linear stress-strain relation is written as

$$\underline{\sigma} = [S]\,\underline{\epsilon} \tag{3.4}$$

where $\underline{\sigma}$ is the stress vector

$\underline{\epsilon}$ is the strain vector

and $[S]$ is a square matrix having 9 components, known as elastic constants. For plane stress problem matrix $[S]$ reduces to :

$$[S] = \begin{bmatrix} E_{11} & E_{12} & 0 \\ E_{12} & E_{22} & 0 \\ 0 & 0 & G_1 \end{bmatrix}$$

Note: $E_{12} = E_{21}$. In engineering practice, material constants used are E_x, E_y, ν_x, ν_y and G. These constants are related to the elements of $[S]$ in the following form :

$$E_{11} = \frac{E_x}{1-\nu_x\nu_y},\; E_{22} = \frac{E_x}{1-\nu_x\nu_y},\; E_{12} = E_{21} = E_{22}\nu_x = E_{11}\nu_y,$$

$$G_1 = G\,. \tag{3.6}$$

The governing equation for free transverse vibrations of an orthotropic plate is

$$D_x\frac{\partial^4 w}{\partial x^4} + 2H\frac{\partial^4 w}{\partial x^2\partial y^2} + D_y\frac{\partial^4 w}{\partial y^4} + \rho_p h\frac{\partial^2 w}{\partial t^2} = 0 \tag{3.7}$$

where $H = D_1 + 2D_{xy}$ and

$$D_x = \frac{E_{11}h^3}{12},\; D_y = \frac{E_{22}h^3}{12},\; D_1 = \frac{E_{12}h^3}{12} = \frac{E_{21}h^3}{12},$$

$$D_{xy} = \frac{Gh^3}{12} \tag{3.8}$$

Let D_o , the flexural rigidity parameter for the orthotropic plate, S.S. on the edges, be defined as

$$D_o \equiv \frac{m^4}{a^4} D_x + \frac{2m^2n^2}{a^2b^2} H + \frac{n^4}{b^4} D_y \qquad (3.9)$$

then the frequency equation for the plate is written as[8]:

$$p_{mn} = \pi^2 \sqrt{\frac{g}{\gamma_p h} D_o} \qquad (3.10)$$

where γ_p is the weight density and h is the thickness of the plate. It is necessary that for finding the frequencies of the given orthotropic grillage, the flexural rigidity parameter D_o, and the thickness h of the equivalent orthotropic plate should be known. To do so proposed plate analogy is used.

(i) <u>Energy expressions for the orthotropic grid work</u> :

The potential and kinetic energy expressions U_g and K_g for the grid work vibrating in m,n mode derived in Appendix-A, are given as :

$$U_g = \frac{1}{8} B_{mn}^2 \pi^4 \sin^2 p_{mn} t \left[m^4 \frac{E_x I_x}{a^3} (N_x+1) + n^4 \frac{E_y I_y}{b^3} (N_y+1) \right] \quad \ldots \qquad (3.11)$$

and

$$K_g = \frac{1}{8} B_{mn}^2 p_{mn}^2 \cos^2 p_{mn} t \left[m_x a(N_x+1) + m_y b(N_y+1) \right] \qquad (3.12)$$

(ii) Energy expression for the orthotropic plate:

Similarly the energy expressions U_p and K_p for the equivalent orthotropic plate in m,n mode are given below, refer Appendix-A,

$$U_p = \frac{1}{8} A_p B_{mn}^2 \pi^4 \sin^2 p_{mn} t \left[\frac{m^4}{a^4} D_x + \frac{n^4}{b^4} D_y + 2\frac{m^2 n^2}{a^2 b^2} H \right] \quad \ldots \quad (3.13.a)$$

which in terms of D_o, defined earlier, is

$$U_p = \frac{1}{8} B_{mn}^2 A_p D_o \pi^4 \sin^2 p_{mn} t \qquad (3.13\text{-}b)$$

and

$$K_p = \frac{1}{8} B_{mn}^2 A_p M_p p_{mn}^2 \cos^2 p_{mn} t \qquad (3.14)$$

According to the proposed analogy, the maximum strain and kinetic energy expressions of the plate are to be equated with the maximum strain and kinetic energy expressions of the given grillage, thus yielding the required unknown parameters of the orthotropic plate. Thus equating U_g, eq. 3.11, with U_p, eq. 3.13, one gets

$$D_o = \frac{1}{A_p} \left[\frac{E_x I_x}{a^3} (N_x+1) m^4 + \frac{E_y I_y}{b^3} (N_y+1) n^4 \right] \qquad (3.15)$$

Similarly equating the kinetic energy of the orthotropic grid K_g, eq. 3.12, to that of the equivalent orthotropic plate K_p eq. 3.14, one obtains the expression

$$M_p A_p = m_x a (N_x+1) + m_y b (N_y+1) \qquad (3.16)$$

Since mass per unit area of the plate M_p is

$$M_p = \rho_p h .$$

h is obtained from 3.16 as

$$h = \frac{1}{\rho_p A_p} \left[m_x\, a\, (N_x+1) + m_y\, b\, (N_y+1) \right] \qquad (3.17)$$

Thus the parameters of the equivalent orthotropic plate D_o and h, are known in terms of the grid parameters m_x, m_y, N_y, N_x, a,b,E_x and E_y, which in turn can be directly used to get the frequencies of the orthotropic grid, eq. 3.10. The following simplified cases of the orthotropic grillage are considered :

(a) Orthotropic grid having beams equally spaced in both directions.

(b) Square orthotropic grid having beams equally spaced in both directions.

(a) <u>Orthotropic grid having same spacing in both directions</u> :

Having found the flexural rigidity parameters D_o and thickness h of the equivalent orthotropic plate, it is easy to get these for special cases. For

$$\frac{a}{N_y+1} = \frac{b}{N_x+1} \qquad (3.18)$$

Using equations (3.15), (3.17) and (3.18) one obtains the two required parameters for the corresponding equivalent orthotropic plate. Thus

$$D_o = \frac{a(N_x+1)}{A_p}\left[\frac{E_x I_x}{a^4} m^4 + \frac{E_y I_y}{b^4} n^4\right] \tag{3.19}$$

and
$$h = \frac{a(N_x+1)}{\rho_p A_p}\left[m_x + m_y\right] \tag{3.20}$$

(b) Square orthotropic grid having beams equally spaced in x and y directions:

In this special case of an orthotropic grid, the sides a and b of the grid are equal and the expressions given by (3.19) and (3.20) reduce to

$$D_o = \frac{N_x+1}{a^5}\left[E_x I_x m^4 + E_y I_y n^4\right] \tag{3.21}$$

and
$$h = \frac{N_x+1}{\rho_p a}\left(m_x + m_y\right) \tag{3.22}$$

Thus the analogy formulation for an orthotropic grillage is complete. To validate this, a number of cases are considered in the next chapter.

B - Isotropic grid :-

An isotropic grid is a special case of the orthotropic grid when the spacing, material and geometric properties of the beams are same in both the directions. Correspondingly an isotropic plate is a special case of orthotropic plate in which material properties have no preferred direction. Thus for an isotropic grid

$$m_x = m_y = m_b,\ I_x = I_b,\ E_x = E_y = E_b \tag{3.23}$$

and

$$\frac{a}{N_y+1} = \frac{b}{N_x+1} , \; D_x = D_y = H = D_i = \frac{E_p h^3}{12(1 - \nu^2)}$$

From (3.15) and (3.17), one thus gets the flexural rigidity D_i and thickness h for the equivalent isotropic plate as

$$D_i = \frac{E_b I_b \; (N_x+1) \left[m^4/a^4 + n^4/b^4 \right]}{b \; (m^2/a^2 + n^2/b^2)^2} \tag{3.24}$$

and

$$h = \frac{2 \; (N_x+1) \; m_b}{\rho_p b} \tag{3.25}$$

The special case of an isotropic grid is that with square boundaries and the above equations reduce to

$$D_i = \frac{E_b I_b \; (N_x+1) \; (m^4+n^4)}{a \; (m^2 + n^2)^2} \tag{3.26}$$

and

$$h = \frac{2 \; (N_x+1) \; m_b}{\rho_p a} \tag{3.27}$$

For a square isotropic grid non-dimensional parameters flexural rigidity p and thickness parameter q are defined as

$$p \equiv \frac{D_i a}{E_b I_b \; N_x} \tag{3.28}$$

$$q \equiv \frac{h \; \rho_p a}{2 \; m_b N_x} \tag{3.29}$$

Substituting expressions for D_i and h from eq. (3.28) and (3.29) in eqs. (3.26) and (3.27) one obtains

$$p = (1 + \frac{1}{N_x}) \; \frac{m^4 + n^4}{(m^2+n^2)^2} \tag{3.30}$$

$$q = (1 + \frac{1}{N_x}) \tag{3.31}$$

Note that nondimensional flexural rigidity parameter p depends upon the order of the mode of vibration whereas the nondimensional thickness parameter q is independent of it.

Similar dimensionless parameters, tension parameter $p_n = Sa/N_x T$ and thickness parameter $q_n = h\, a^2/2A_c a N_x$ were defined for cable network[1], where

A_c = Area of cross-section of each cable.

S = Tension per unit length of the equivalent membrane

T = Tension in each cable.

These parameters were found to be equal and given by[1]

$$p_N = q_N = 1 + 1/N_x \qquad (3.31\text{b})$$

3.1.3 <u>Grillages clamped on the boundaries</u>:

An orthotropic plate fixed on the edges is not exactly solvable but approximate methods have been used for its solution. Raleigh method has been used for rectangular orthotropic plate whereas results for square isotropic plate are known using Raleigh-Ritz method. For establishing analogy, expression for frequency derived by Raleigh's energy method is used here.

The admissible function used for plate analysis is obtained from beam functions $X_m(x)$, $Y_n(y)$ of the grillage and the displacement function w(x,y,t) is taken as[27]

$$w(x,y,t) = \sum_{m=1}^{p} \sum_{n=1}^{q} B_{mn}\, X_m(x)\, Y_n(y) \sin p_{mn} t \qquad (3.32)$$

The beam functions[27] (eigen function) for clamped-clamped beam are

$$X_r(x) = \text{Cosh}\,\frac{e_r x}{a} - \text{Cos}\,\frac{e_r x}{a} - g_r\left(\text{Sinh}\,\frac{e_r x}{a} - \text{Sin}\,\frac{e_r x}{a}\right) \quad (3.33)$$

$$r = 1, 2, \ldots\ldots$$

where e_r g_r are constants and their numerical values for each set of functions are given in Table-A.

TABLE-A

r	gr	e_r
1	0.9825	4.73
2	1.0007	7.8532
3	0.5359	10.9956
4	1.000	14.1371
5	0.9999	17.2787
6	1.0000	20.4203
r > 6	1	$(2r+1)\pi/2$

In Raleigh's energy method, one is mainly interested in fundamental frequency and uses only $X_1(x)$ and $Y_1(y)$. The admissible function for the plate, clamped on the edges, can thus be written, using eqs. (3.32) and (3.33), as

$$w\,(x,y,t) = B_{mn}\left[\text{Cosh}\,\frac{e_m x}{a} - \text{Cos}\,\frac{e_m x}{a} - g_m\left(\text{Sinh}\,\frac{e_m x}{a} - \text{Sin}\,\frac{e_m x}{a}\right)\right]\left[\text{Cosh}\,\frac{e_n y}{b} - \text{Cos}\,\frac{e_n y}{b} - g_n\left(\text{Sinh}\,\frac{e_n y}{b} - \text{Sin}\,\frac{e_n y}{b}\right)\right]\text{Sin}\,p_{mn}t \quad (3.34)$$

Hearmon[8] using Raleigh energy method gives the expression for the fundamental frequency of the orthotropic plate, clamed on the edges, as

$$p_{mn} = \sqrt{\frac{1}{\rho_p h}\left[D_x \frac{e_m^4}{a^4} + D_y \frac{e_n^4}{b^4} + 2H \frac{e_m e_n}{a^2 b^2}(e_m - 2)(e_n - 2)\right]} \quad \ldots\ldots \quad (3.35)$$

for, m = n = 1.

Let the flexural rigidity parameter D_{oF} for such a plate be defined as

$$D_{oF} = D_x \frac{e_m^4}{a^4} + D_y \frac{e_n^4}{b^4} + 2H \frac{e_m e_n}{a^2 b^2}(e_m - 2)(e_n - 2) \qquad (3.36)$$

Then the frequency equation can be rewritten as

$$p_{mn} = \sqrt{D_{oF}/\rho_p h} \quad \text{for} \quad m = n = 1 \qquad (3.37)$$

A special case of the clamped-clamped orthotropic plate is an isotropic plate, clamed on the edges, for which Young[27] found frequencies in different modes using Raleigh Ritz method. The calculation were carried out for a 36 term series based on taking both m and n equal to 1,2,3,4, 5 and 6. Young's results for a square isotropic plate are reproduced in Table B.

TABLE-B

Mode No.	1	2	3	4	5	6
$p_{mn}/\sqrt{\frac{D_i}{\rho_p h a^4}}$	35.99	73.41	108.27	131.64	132.25	165.[illegible]

The rectangular isotropic plate has been solved by various authers[2, 12, 14, 17] and their results (upto fifth frequency) are shown in the figure 3.3. The value of the parameter $p_{mn} b^2 \sqrt{\rho h_p / D_i}$, D_i being the flexural rigidity parameter for isotropic plate can be read directly for a known a/b ratio.

A Orthotropic grids:

Proceeding in the same way as for simply supported case, the energy expressions for the grid and plate are found. Strain energy U_g and kinetic energy K_g of the grid are derived and their expressions are

$$U_g = \tfrac{1}{4} B_{mn}^2 \sin^2 p_{mn} t \left[E_x I_x (e_m/a)^3 G_m H_n^{N_x} + E_y I_y (e_n/b)^3 G_n H_m^{N_y} \right] \tag{3.38}$$

and

$$K_g = \tfrac{1}{4} B_{mn}^2 p_{mn}^2 \cos^2 p_{mn} t \left[\frac{m_x a}{e_m} G1_m H_n^{N_x} + \frac{m_y b}{e_n} G1_n H_m^{N_y} \right] . \tag{3.39}$$

Whereas strain energy[8] U_p and kinetic energy K_p of the plate are

$$U_p = B_{mn}^2 \frac{ab}{2} \sin^2 p_{mn} t \left[D_x \frac{e_m^4}{a^4} + D_y \frac{e_n^4}{b^4} + \frac{2 H e_m e_n (e_m - 2)(e_n - 2)}{a^2 b^2} \right] \tag{3.40}$$

where, $H = D_1 + 2D_{xy}$

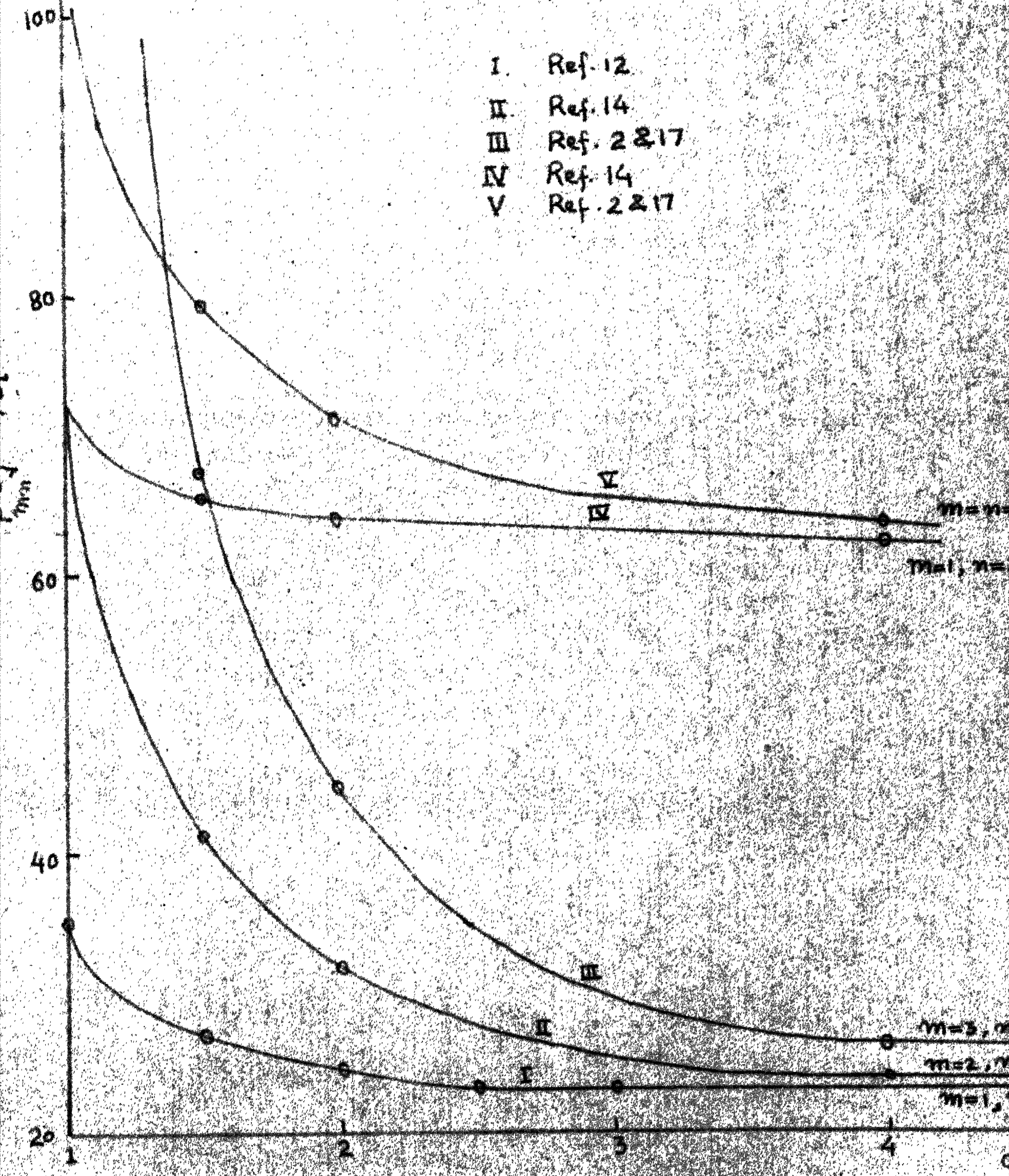


Fig. 3-3. Graph Showing $P_{mn} b^2 \sqrt{\rho_p h / D_1}$ Vs. a/b

and

$$K_p = \frac{1}{8} M_p p_{mn}^2 B_{mn}^2 \cos^2 p_{mn} t \frac{ab}{e_m e_n} G1_m G1_n ; \quad (3.41)$$

the constants $H_n^{N_x}$, $H_m^{N_y}$, G_m , $G1_m$ are given below.

$$H_m^{N_x} = \sum_{j=1}^{N_y} \left[\text{Cosh} \frac{e_m}{N_y+1} j - \text{Cos} \frac{e_m}{N_y+1} j - g_m \left(\text{Sinh} \frac{e_m}{N_y+1} j - \text{Sin} \frac{e_m j}{N_y+1} \right) \right]^2 \quad (3.42)$$

$$H_n^{N_y} = \sum_{i=1}^{N_x} \left[\text{Cosh} \frac{e_n i}{N_x+1} - \text{Cos} \frac{e_n i}{N_x+1} - g_n \left(\text{Sinh} \frac{e_n i}{N_x+1} - \text{Sin} \frac{e_n i}{N_x+1} \right) \right]^2 \quad (3.43)$$

$$G_r = (1+g_r^2)(0.5 \text{ Sinh } 2 e_r + 2 \text{ Cosh } e_r \text{ Sin } e_r) + (1-g_r^2)(0.5 \text{ Sin } 2 e_r + 2 \text{ Sinh } e_r \text{ Cos } e_r) + 2 e_r - g_r (\text{Cosh } 2 e_r + 4 \text{ Sin } e_r \text{ Sinh } e_r - \text{Cos } 2 e_r) \quad \ldots (3.44)$$

$$G1_r = (1+g_r^2)(0.5 \text{ Sinh } 2 e_r - 2 \text{ Cosh } e_r \text{ Sin } e_r) + (1-g_r^2)(0.5 \text{ Sin } 2 e_r - 2 \text{ Sinh } e_r \text{ Cos } e_r) + 2 e_r - g_r (\text{Cosh } 2 e_r - 4 \text{ Sin } e_r \text{ Sinh } e_r - \text{Cos } 2 e_r) \quad \ldots (3.45)$$

r takes all the integervalues of m and n.

The constants G_r , $G1_r$ are calculated for various values of r and the results are given in Table 1. It is to be noted

that these do not depend upon the order of the grid that is number of beams N_x or N_y, in the grid. Constants $H_n^{N_x}$ and $H_m^{N_y}$ are calculated for various order grids and depend upon N_x or N_y. Their values are given in Table-2. Replacing expression in the brackets of equation (3.40) by D_{OF}

$$U_p = \tfrac{1}{2}\, ab\, D_{OF}\, B^2 \, Sin^2\, p_{mn} t \qquad (3.46)$$

Equating maximum strain and kinetic energy expressions of the grid and the plate one gets the flexural rigidity parameter D_{OF} and the thickness h, of the equivalent orthotropic plate and these are

$$D_{OF} = \frac{1}{2ab} \left[E_x I_x \left(\frac{e_m}{a}\right)^3 G_m H_n^{N_x} + E_y I_y \left(\frac{e_n}{b}\right)^3 G_n H_m^{N_y} \right] \qquad (3.47)$$

and

$$h = \frac{2\, e_m e_n}{\rho_p ab\; G1_m\, G1_n} \left[\frac{m_x a}{e_m} G1_m H_n^{N_x} + \frac{m_y b}{e_n} G1_n H_m^{N_y} \right] \qquad (3.48)$$

To know the equivalent plate completely, one needs expressions for D_x, D_y and H. But here - one expression eq. (3.36) is available which only correlates these constants therefore it is not possible to determine the values of these constants individually. However, as for as the problem of determining fundamental frequency is concerned one needs expression for D_{OF} as a whole. Thus it is sufficient to calculate value of D_{OF} only. A special case of a clamped rectangular orthotropic grid is a clamped square orthotropic grid for which the required parameters are given as :

$$D_{oF} = \frac{1}{2a^5}\left[E_x I_x\, e_m^3\, G_m\, H_n^{N_x} + E_y I_y\, e_n^3\, G_n\, H_m^{N_y}\right] \qquad (3.49)$$

and

$$h = \frac{2\, e_m e_n}{\rho_p a\, G1_m\, G1_n}\left[\frac{m_x\, G1_m\, H_n^{N_x}}{e_m} + \frac{m_y\, G1_n\, H_m^{N_y}}{e_n}\right] \qquad (3.50)$$

B <u>Isotropic Grids</u>:

An isotropic grid is a special case of an orthotropic grid and for this the flexural rigidity parameter D_i and thickness h are directly obtained from expressions (3.36), (3.47) and (3.48)

$$D_i = \frac{\frac{E_b I_b}{2\,ab}\left[\left(\frac{e_m}{a}\right)^3 G_m\, H_n^{N_x} + \left(\frac{e_n}{b}\right)^3 G_n\, H_m^{N_y}\right]}{\left[\frac{e_m^4}{a^4} + \frac{e_n^4}{b^4} + \frac{2\, e_m e_n}{a^2 b^2}(e_m - 2)(e_n - 2)\right]} \qquad (3.51)$$

and

$$h = \frac{2mb}{\rho_p}\left[\frac{e_n\, H_n^{N_x}}{b\, G1_n} + \frac{e_m\, H_m^{N_y}}{a\, G1_m}\right] \qquad (3.52)$$

In case of a clamped square isotropic grid, these parameters take the form :

$$D_i = \frac{E_b I_b}{2\,a}\left[\frac{e_m^3\, G_m\, H_n^{N_x} + e_n^3\, G_n\, H_m^{N_y}}{e_m^4 + e_n^4 + 2\, e_m\, e_n\,(e_m - 2)(e_n - 2)}\right] \qquad (3.53)$$

and

$$h = \frac{2m_b}{a\rho_p}\left[\frac{e_n\, H_n^{N_x}}{G1_n} + \frac{e_m\, H_m^{N_y}}{G1_m}\right] \qquad (3.54)$$

Thus the desired unknown parameters of the equivalent plate are known. The frequency calculated from these, give the frequency of the grid.

3.2 SUMMARY OF RESULTS:

The analogy proposed states that the frequency for the grillage is obtained by substitution of flexural rigidity parameter and thickness of the equivalent plate in the frequency equation of the analogous plate. The results obtained for simply supported and clamped grillages are summarised below:

A - Frequency equation for grids S.S. on the edges:

As per the proposition simply supported grid is to be replaced by an equivalent plate with same boundary conditions. The proposition leads to frequency equation due to the fact that exact solution of plate with simply supported edges is known. For grid with different spacing and material properties in x and y directions, frequency equation is

$$p_{mn} = \pi^2 \sqrt{\frac{\frac{E_x I_x}{a^3}(N_x+1)m^4 + \frac{E_y I_y}{b^3}(N_y+1)n^4}{m_x a\,(N_x+1) + m_y b\,(N_y+1)}} \qquad (3.55)$$

For orthotropic grids having uniform beam spacing in both x and y-direction, above equation reduces to :

$$p_{mn} = \pi^2 \sqrt{\frac{\frac{E_x I_x}{a^4} m^4 + \frac{E_y I_y}{b^4} n^4}{(m_x + m_y)}} \qquad (3.56)$$

For a rectangular isotropic grid the frequency equation becomes:

$$p_{mn} = \pi^2 \sqrt{\frac{E_b I_b}{m_b} \left[\frac{m^4}{a^4} + \frac{n^4}{b^4} \right]} \tag{3.57}$$

For square isotropic grid, S.S. on the edges, frequency equation reduces to

$$p_{mn} = \pi^2 \sqrt{\frac{E_b I_b}{2 m_b a^4} (m^4 + n^4)} \tag{3.58-a}$$

or

$$p_{mn} = \pi^2 \sqrt{\frac{E_b I_b}{2 \rho_b A_b a^4} (m^4 + n^4)} \tag{3.58-b}$$

B <u>Grillages clamped on the edges</u> :

As mentioned earlier, no closed form solution is known for the frequencies of plates clamped on the edges. However equation (3.37) can be used to give an approximate result for the fundamental frequency of an orthotropic plate. Thus the fundamental frequency of the orthotropic grids is obtained by substitution of D_{OF} and h (equations 3.47 and 3.48) in equation 3.37.

$$p_{mn} = \sqrt{\frac{G1_m \, G1_n}{4 e_m \, e_n} \left[\frac{E_x I_x \left(\frac{e_m}{a}\right)^3 G_m \, H_n^{N_x} + E_y I_y \left(\frac{e_n}{b}\right)^3 G_n \, H_m^{N_y}}{\frac{m_x a \, G1_m \, H_n^{N_x}}{e_m} + \frac{m_y b \, G1_n \, H_m^{N_y}}{e_n}} \right]}$$

$m = n = 1$,

..... (3.59)

or

$$p_{11} = \sqrt{\frac{G1_1\ G_1\ e_1{}^2}{4}\left[\frac{E_xI_x(1/a)^3\ H_1{}^{N_x} + E_yI_y(1/b)^3\ H_1{}^{N_y}}{m_xa\ H_1{}^{N_x} + m_yb\ H_1{}^{N_y}}\right]}$$

For a square orthotropic grid, the fundamental frequency reduced to

$$p_{11} = \sqrt{\frac{G1_1\ G_1\ e_1{}^2}{4\ a^4}\left[\frac{E_xI_xH_1{}^{N_x} + E_yI_y\ H_1{}^{N_y}}{m_x\ H_1{}^{N_x} + m_y\ H_1{}^{N_y}}\right]} \tag{3.60}$$

For isotropic plates several authors[2,12,14,17,27] have calculated the value of the parameter f_{mn} defined as

$$f_{mn} = \frac{p_{mn}}{\sqrt{\frac{D_i}{\rho_p\ h\ b^4}}} \tag{3.61}$$

$m,n = 1,2 \ldots,$

for various modes and for various side ratios a/b of the plate. The flexural rigidity parameter D_i and the thickness h, of the equivalent plate, derived in equations 3.51 and 3.52, are substituted in eq. 3.61 so as to give the frequencies of the grid, clamped on the edges.

The frequencies of the grillages are obtained by proposing the plate analogy. This proposition needs validation. The proposition will be verified by taking various case studies. The details of the procedure are given in the next chapter.

CHAPTER-IV

CASE STUDIES

In chapter III, frequency equations for grids with two different boundary conditions, simply supported and clamped, have been derived from the proposition of the plate analogy. To validate the proposition, a number of grillages, mentioned below, have been solved by the finite element method and the results are compared with that of the proposed equivalent plate.

A <u>Grids, simply supported</u>:

(i) Orthotropic grids

(a) Rectangular

(b) Square

(ii) Isotropic grids

(a) Rectangular

(b) Square

B <u>Grids clamped:</u>

(i) Orthotropic grids

(a) Rectangular

(b) Square

(ii) Isotropic grids

(a) Rectangular

(b) Square

General treatment for cases (A) and (B ii) is given whereas case (B i) is dealt for its fundamental frequency only.

4.1 FINITE ELEMENT METHOD :

The Finite Element method is a well known technique for structural analysis and is not discussed in details. However its relevence to the present problem is discussed in the following steps:

4.1.1 Basic element of the grid

The beam element between the two joints of the grid is taken as the basic element. As mentioned earlier, only bending vibration of the grillage have been considered and therefore the basic element of the grid has four degrees of freedom shown in figure 4-1. Out of these two degrees of freedom are associated with the transverse deflection (V_1 and V_2) whereas the remaining two are associated with the rotations (θ_1 and θ_2).

4.1.2 Numbering system of the joints:

To denote the degrees of freedom some numbers have been assigned to these. The numbering system for

the grid, S.S. on the boundaries, is shown in figure 4-2 and for the grid, clamped on the edges, is shown in figure 4-3. Each internal joint is identified by a displacement coordinate and two rotations in xz and yz planes. Thus each internal joint is represented by three numbers of which,

(1) First indicates the degree of freedom associated with the transverse deflection.

(2) Second indicates the degree of freedom associated with the slope of the beam in x-z plane.

(3) Third denotes the degree of freedom associated with the slope of the beam in y-z plane.

In case of S.S. grids (Fig. 4-2), the joints on the boundaries have only one degree of freedom associated with the rotation whereas the joints of the grids clamped on the boundaries have no degree of freedom.

4.1.3 Stiffness and Mass matrices of the grid:

The mass and stiffness matrices $[M]$ and $[K]$ respectively of the grid is obtained by synthesis of mass and stiffness of the constituting element as per standard procedure. Let the mass and stiffness matrix of the element be $[m]$ and $[k]$, then the force deflection relation[14] is written as

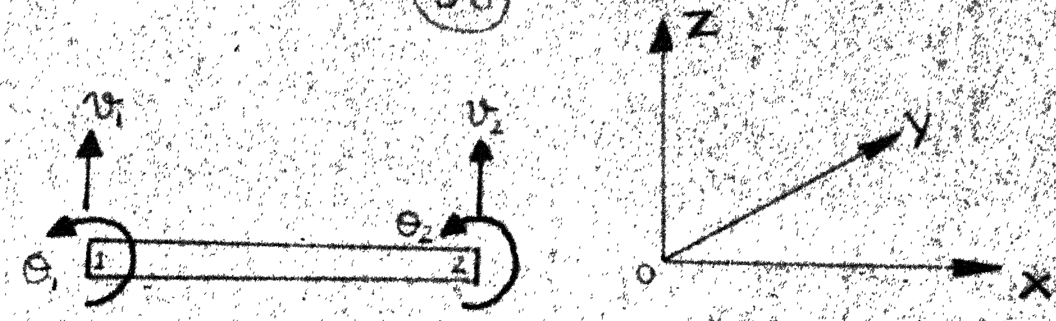
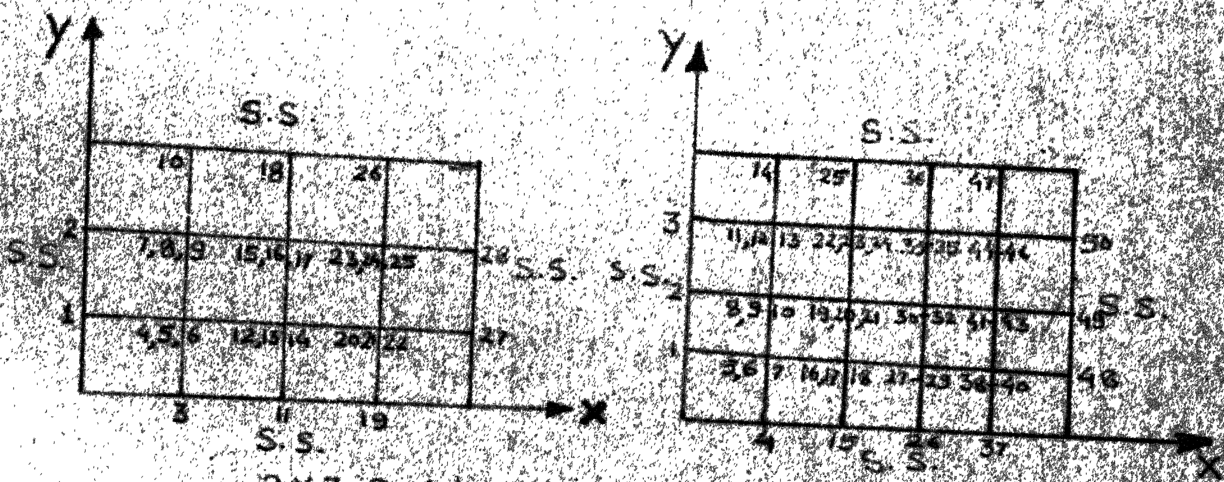


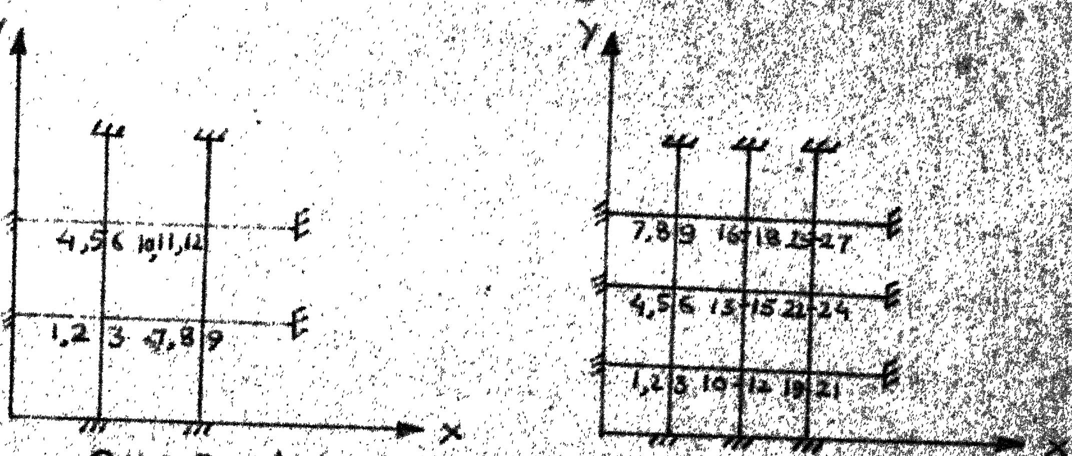
Fig. 4-1. Grid Element



2X3 Grid

3X4 Grid

Fig. 4-2. Numbering System of Joints in S.S. Grids



2×2 Grid

3×3 Grid

Fig. 4-3. Numbering System of Joints in Clamped Grids.

$$\begin{Bmatrix} S_1 \\ M_1 \\ S_2 \\ M_2 \end{Bmatrix} = \begin{bmatrix} k \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} \tag{4.1}$$

where S_i and M_i are shear forces and bending moments at the joint i and

$$[k] = \frac{E_b I_b}{l^3} \begin{array}{c} \begin{array}{cccc} v_1 & \theta_1 & v_2 & \theta_2 \end{array} \\ \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \end{array} \tag{4.2}$$

Correspondingly one can write the mass matrix (with cubic displacement curve[13]) for the element as

$$[m] = \frac{m_b l}{420} \begin{array}{c} \begin{array}{cccc} v_1 & \theta_1 & v_2 & \theta_2 \end{array} \\ \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22 & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \end{array} \tag{4.3}$$

l being the length of the basic element.

4.1.4 Determination of frequencies:

The differential equation of free oscillation of the discretized system is

$$[M]\{\ddot{q}\} + [K]\{q\} = 0 \tag{4.4}$$

Having obtained $\left[M\right]$ and $\left[K\right]$, one solves the equation (4.4) for its eigen values and corresponding eigenvectors by Matrix Iteration method[11].

A computer programm, refer Appendix C, is given to determine the frequencies and mode shapes of the rectangular orthotropic grid. The programm furnishes as many eigenvalues as desired but gives mode shapes upto second only.

4.2 LIST OF EXAMPLES :

The list of grids considered for the dynamic analysis is given in Table C. The frequencies of all these grids are given in Appendix-B. The mode shapes (upto 2nd mode) have also been found for all these cases.

Define non dimensional frequency parameter

$$\mathcal{U} = \left[\frac{p_{mn}^2 \quad A_b \, \rho_b \; a_1}{E_b I_b}\right]^{0.25} \qquad (4.5)$$

where a_1 is the spacing of the beams. It is calculated for 3 x 3 isotropic grid from the proposed analogy method and its results are compared with those obtain from earlier methods[3,4,6,7,23], refer Table-3.

Table Shows kind, order and Geomatrical and Material Propeties of the Grids.

S.No.	Type of the Grid	Order	a in.	a/b	Material Properties of beams: E_x psi	E_y psi	γ_x lb	γ_y lb	Geometrical Properties of beams: A_x in^2	A_y in^2	I_x in^4	I_y in^4
1		2x3		1.332								
2	Orthotropic	3x4	50	1.25	30×10^6	30×10^6	0.284	0.284	3.0	1.875	0.25	0.088
3		2x3		1.332								
4	Grids	3x4	50	1.25	30×10^6	12×10^6	0.284	0.1	3.0	3.0	0.25	0.25
5	S.S. on	2x2										
6		3x3	50	1	30×10^6	30×10^6	0.284	0.284				
7	the edges.	4x4							3.0	1.875	0.25	0.088
8		2x2										
9		3x3 4x4	50	1	30×10^6	12×10^6	0.284	0.1				
10									3.0	3.0	0.25	0.25
11	Isotropic	2x3	50									
12	Grids	3x4		1.332 1.25								
13	S.S. on the	2x2			30×10^6	30×10^6	0.284	0.284	3.0	3.0	0.25	0.25
14	edges	3x3	50	1								
15		4x4										
16	Orthotropic	2x3		1.332								
17	Grids	3x4	50	1.25								
18	clamped	4x5		1.2								
19	on the edges.	2x2			30×10^6	30×10^6	0.284	0.284	3.0	1.875	0.25	0.088
20	[Fundamental	3x3	50	1								
21	frequency	4x4										
22	only]	5x5										
23		2x3		1.332								
24	Isotropic	3x4	50	1.25								
25	Grids	4x5		1.20								
26	clamped	2x2			30×10^6	30×10^6	0.284	0.284	3.0	3.0	0.25	0.25
27	on	3x3										
28	the edges.	4x4	50	1								
29		5x5										

4.3 DISCUSSION OF THE RESULTS AND VALIDATION OF THE PROPOSED ANALOGY:

The results of frequencies of different grids considered in the present work, refer Table-C, are given in Appendix-B. For simply supported grids, Tables 6, 7 and 9, it is clear that for a square grid of specified dimensions and beam properties, the natural frequencies converge to those of the equivalent plate as the order of the grid increases. This implies that the equivalent plate for all the grids of different orders with same dimensions and beam properties is a unique one.

It is also observed that in some cases the results obtained by Finite Element method are higher than by proposed analogy whereas in other cases these are lower (Refer Tables 4 to 9). One of the reason for the erratic performance of the Finite Element method in these examples is the fact that compatibility conditions are not satisfied at the joints (Assumption of torsionless grid). The lack of compatibility induces excessive flexibility, while the assumed deformation pattern represent constraints which lead to excessive stiffness of the individual elements. The combination of elements is either too flexible or too stiff depending upon which effect predominates.

The results obtained for different grids, given in Appendix-B, clearly show that the frequencies obtained by the proposed analogy are comparable with those obtained

by Finite Element method. Also table-3 shows the closeness of the nondimensional frequency parameter u found by the proposed analogy method to that obtained earlier by other authors[3,4,6,7,23].

Timoshenko proposed intutively the analogy of an orthotropic plate for static analysis of the beam grillage. According to his analogy, mass and flexural rigidity of the beams are assumed to be evenly distributed over the pitch of the grid. The proposition thus can be extended for dynamic analysis. Frederick and Falgout[7] calculated the nondimensional frequency parameter based upon orthotropic plate theory and showed closeness of his results to those obtained by discrete analysis. The results obtained in the present work are equally close to those of Finite Element method and those obtained by earlier research workers. However unlike Timoshenko's proposition of analogy based on intution, the present analogy is based on the rationale of energy equivalence and holds good for grids with any number of beams.

Based upon Timoshenko's proposition following relations for the analysis of a torsionless isotropic grid are obtained.

$$D_i = \frac{E_b I_b}{a} (N_x+1) \tag{4.6}$$

and

$$h = \frac{2 \rho_b A_b N_x}{a \rho_p} \tag{4.7}$$

Renton[19] derived equlibrium equation for static analysis of an orthotropic grid as

$$\frac{E_x I_x}{b_1}\frac{\partial^4 w}{\partial x^4} + \left(\frac{Gj_x}{b_1} + \frac{Gj_y}{a_1}\right)\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{E_y I_y}{a_1}\frac{\partial^4 w}{\partial y^4} = q_{xy} \qquad (4.8)$$

For a torsionless isotropic square grid, contribution due to torsional rigidities are neglected and the equation reduces to

$$\frac{E_b I_b}{a_1}\left(\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4}\right) = q_{xy} \qquad (4.9)$$

Timoshenko's proposition gives the following form of equation of motion for transverse vibration of the equivalent isotropic square plate

$$D_i\left(\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4}\right) = -\rho_p h\frac{\partial^2 w}{\partial t^2} \qquad (4.10)$$

The frequency equation of (4.10) for simply supported boundary condition is

$$p_{mn} = \pi^2\sqrt{\frac{D_i}{\rho_p h\, a^4}\,(m^4 + n^4)} \qquad (4.11)$$

Substituting D_i and h from eq. (4.6) and (4.7) in eq. (4.11), one gets the frequency equation for a torsionless square isotropic grid, simply supported at the edges as

$$p_{mn} = \pi^2\sqrt{\frac{E_b I_b}{2\rho_b A_b a^4}\left(1+\frac{1}{N_x}\right)(m^4 + n^4)} \qquad (4.12)$$

Note that as N_x is very large the frequency equation (4.12), approaches to that obtained by proposed analogy, refer eq. (3.58-b).

In the present formulation of plate analogy it has been assumed that the equivalent plate behaves as a real plate whereas Timoshenko's proposition **violates** the real model of the plate in bending, refer eq. (4.10). Note eq. (4.10) is not of the form of plate in transverse vibration.

CHAPTER V

CONCLUSION

In the present work a plate analogy of the torsionless beam grillage for its dynamic analysis has been proposed. The analogy has been proposed to arrive at simple procedure for dynamic analysis of the grids because discrete approach, such as Finite Element method, is time consuming. The analogy proposed in this work is presented on the rationale of energy equivalence whereas Timoshenko's orthotropic plate analogy is proposed on the intution that for very large order of the beam grillage it approaches an orthotropic plate.

The proposition of the plate analogy has been validated by comparing the results of the frequencies obtained by the proposed analogy method with those found by Finite Element method. These results also compare well with those obtained by earlier research workers. Note that the analogy proposition is easily formulated for the grillage with simply supported edges due to the fact that the equivalent orthotropic plate with S.S. edges is exactly solvable. For orthotropic beam grillage with fixed ends frequency equations for 1st mode is derived using admissible function and the proposition is validated. However

for isotropic beam grillage, clamped on the edges, results are obtained for higher modes as well which compare reasonably with Finite Element method.

SCOPE FOR FURTHER WORK:

Since the analogy proposition simplifies the method for dynamic analysis, it is proposed that further work should be done to validate the analogy based on energy equivalence to the cases of grids with torsion. It will be quite interesting to find out how far a proposition can be formulated for non orthogonal beam grillages which may be more of academic interest then of engineering applications.

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APPENDIX–A

A-a Energy expressions for orthotropic orthogonal grid work, S.S. on all edges :

Taking the modal functions of beam to be obtained from that of plate at known x and y, as mentioned in chapter 3, the mode shape for ith beam lying in x direction, can be written as (Refer eq. 3.2).

$$w_i = B_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi i}{N_x+1} \sin p_{mn}t \qquad \text{(A-1)}$$

The strain energy of a beam in bending is written as

$$U_b = \frac{E_b I_b}{2} \int_0^l \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \qquad \text{(A-2)}$$

Therefore the strain energy of the ith beam lying in x direction is

$$U_{xi} = \frac{E_x I_x}{2} \int_0^a \left(\frac{\partial^2 w_i}{\partial x^2}\right)^2 dx \qquad \text{(A-3)}$$

From equations (A-1) and (A-3)

$$U_{xi} = \frac{E_x I_x}{2} \int_0^a \left[B_{mn}\left(\frac{m\pi}{a}\right)^2 \sin \frac{n\pi i}{N_x+1} \sin \frac{m\pi x}{a} \sin p_{mn}t\right]^2 dx$$

$$= \frac{E_x I_x}{2} B_{mn}^2 \left(\frac{m\pi}{a}\right)^4 \sin^2 \frac{n\pi i}{N_x+1} \sin^2 p_{mn}t \int_0^a \sin^2 \frac{m\pi x}{a}\, dx$$

or

$$U_{xi} = B_{mn}^2 \frac{E_x I_x}{2} \left(\frac{m\pi}{a}\right)^4 \sin^2 \frac{n\pi i}{N_x+1} \sin^2 p_{mn}t \left[a/2\right] \quad (A-4)$$

The strain energy U_{xb} of all beams lying along x-axis is obtained by summing the above expression over i, from i=1 to N_x

$$U_{xb} = \frac{E_x I_x}{2} B_{mn}^2 \left(\frac{m\pi}{a}\right)^4 \left[a/2\right] \sin^2 p_{mn}t \sum_{i=1}^{N_x} \sin^2 \frac{n\pi i}{N_x+1}$$

or

$$U_{xb} = \frac{E_x I_x}{2} B_{mn}^2 \left(\frac{n\pi}{a}\right)^4 (a/2) \left[\frac{N_x+1}{2}\right] \sin^2 p_{mn}t \quad (A-5)$$

Similarly one can write the strain energy U_{yb} of all beams lying along y-axis. Thus

$$U_{yb} = \frac{E_y I_y}{2} B_{mn}^2 \left(\frac{m\pi}{b}\right)^4 (b/2) \left[\frac{N_y+1}{2}\right] \sin^2 p_{mn}t \quad (A-6)$$

Therefore the total strain energy U_g, of the orthotropic grillage is

$$U_g = U_{xb} + U_{yb} \quad (A-7)$$

or

$$U_g = \frac{B^2{mn}}{8} \pi^4 \sin^2 p_{mn}t \left[m^4 \frac{E_x I_x}{a^3} (N_x+1) + n^4 \frac{E_y I_y}{b^3} (N_y+1) \right] \quad \ldots \quad (A-8)$$

The kinetic energy expression for the transverse vibration of a beam is given as[11]

$$K_b = \frac{1}{2} \int_0^l m_b \dot{w}^2 \, dx \quad \text{where} \quad \dot{w} = \partial w/\partial t \quad (A-9)$$

Therefore the kinetic energy K_{xi} of ith beam lying in x-direction is

$$K_{xi} = \frac{1}{2} \int_0^a m_x \dot{w}^2 \, dx \qquad \text{(A-10)}$$

Substituting $\dot{w}$ from (A-1) in above expression, one gets

$$K_{xi} = \frac{1}{2} \int_0^a m_x \, p_{mn}^2 \, B_{mn}^2 \, \sin^2 \frac{m\pi x}{a} \, \sin^2 \frac{n\pi i}{N_x+1} \, \cos^2 p_{mn} t \, dx \qquad \text{.. (A-11)}$$

Since m_x, the mass/unit length of the beam, is constant, therefore,

$$K_{xi} = \frac{m_x}{2} \, p_{mn}^2 \, B_{mn}^2 \, \sin^2 \frac{n\pi i}{N_x+1} \, \cos^2 p_{mn} t \int_0^a \sin^2 \frac{m\pi x}{a} \, dx \qquad \text{... (A-12)}$$

Further simplification gives:-

$$K_{xi} = \frac{m_x}{2} \, p_{mn}^2 \, B_{mn}^2 \, \sin^2 \frac{n\pi i}{N_x+1} \cos^2 p_{mn} t \, \left[a/2\right] \qquad \text{(A-13)}$$

Thus the kinetic energy K_{xb} of all beams, in x-direction:

$$K_{xb} = \sum_{i=1}^{N_x} K_{xi} = \frac{m_x}{2} \, p_{mn}^2 \, B_{mn}^2 \, \left[a/2\right] \cos^2 p_{mn} t \sum_{i=1}^{N_x} \sin^2 \frac{n\pi i}{N_x+1}$$

or

$$K_{xb} = \frac{m_x}{2} \, p_{mn}^2 \, B_{mn}^2 \, \left[a/2\right] \, \cos^2 p_{mn} t \left[\frac{N_x+1}{2}\right] \qquad \text{(A-14)}$$

Similarly, one can write the kinetic energy K_{yb} of all beams lying in x-direction. Thus

$$K_{yb} = \frac{m_y}{2} \, p_{mn}^2 \, B_{mn}^2 \left[b/2\right] \, \cos^2 p_{mn} t \left[\frac{N_y+1}{2}\right] \qquad \text{(A-15)}$$

Therefore the total kinetic energy K_g of the orthotropic grillage is obtained by adding expressions (A-13) and (A-14) :-

$$K_g = K_{bx} + K_{by} \qquad (A-16)$$

Hence

$$K_g = \frac{1}{8} p_{mn}^2 B_{mn}^2 \cos^2 p_{mn} t \left[m_x a (N_x + 1) + m_y b (N_y + 1) \right] \quad \cdots \quad (A-17)$$

A-b <u>Energy expressions for the orthotropic plate, S.S. on boundaries:-</u>

Strain energy expression for the bending of orthotropic plate is given by[24]:

$$U_p = \tfrac{1}{2} \int_0^a \int_0^b \left[D_x \left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2D_1 \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + D_y \left(\frac{\partial^2 w}{dy^2}\right)^2 + 4D_{xy} \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \right] dx dy \qquad (A-18)$$

Substituting w, from (A-1) in (A-18) one gets:

$$U_p = \tfrac{1}{2} \int_0^a \int_0^b \Big[D_x B_{mn}^2 \left(\frac{m\pi}{a}\right)^4 \sin^2 \frac{m\pi x}{a} \sin^2 \frac{n\pi y}{b} \sin^2 p_{mn} t + D_y B_{mn}^2 \left(\frac{n\pi}{b}\right)^4 \sin^2 \frac{m\pi x}{a} \sin^2 \frac{n\pi y}{b} \sin^2 p_{mn} t + 2H B_{mn}^2 m^2 n^2 \frac{\pi^4}{a^2 b^2} \sin^2 p_{mn} t \cos^2 \frac{m\pi x}{a} \cos^2 \frac{n\pi y}{b} \Big] dx dy \quad \cdots \quad (A-19)$$

where, $H = D_1 + 2D_{xy}$

or

$$U_p = \frac{1}{8} B_{mn}^2 \pi^4 \operatorname{Sin}^2 p_{mn}t \left[m^4 \frac{D_x}{a^4} (ab) + n^4 \frac{D_y}{b^4} (ab) + 2\, m^2\, n^2 \, \frac{H}{a^2 b^2} (ab) \right] \tag{A-20}$$

Since $ab = A_p$ = Area of the plate

Hence

$$U_p = \frac{1}{8} A_p B_{mn}^2 \pi^4 \operatorname{Sin}^2 p_{mn}t \left\{ D_x \frac{m^4}{a^4} + D_y \frac{n^4}{b^4} + 2H \frac{m^2 n^2}{a^2 b^2} \right] \tag{A-21}$$

The kinetic energy expression for the transverse vibration of the plate is given below[11]

$$K_p = \frac{1}{2} \int_0^a \int_0^b M_p \dot{w}^2 \, dxdy \tag{A-22}$$

Since M_p, the mass per unit area of the plate is constant Therefore:

$$K_p = \frac{M_p}{2} \int_0^a \int_0^b B_{mn}^2 \, p_{mn}^2 \operatorname{Sin}^2 \frac{m\pi x}{a} \operatorname{Sin}^2 \frac{n\pi y}{b} \operatorname{Cos}^2 p_{mn}t \, dxdy \quad \ldots \text{(A-23)}$$

or

$$K_p = \frac{M_p ab}{8} B_{mn}^2 \, p_{mn}^2 \operatorname{Cos}^2 p_{mn}t \tag{A-24}$$

APPENDIX - B

TABLES OF THE RESULTS

TABLE-1

Values of the constants G_r and $G1_r$ for the grids clamped on the edges.

r	G_r	$G1_r$
1	9.46	9.46
2	15.62	15.62
3	29.87	29.87
4	27.27	27.27

TABLE-2

Values of the constant $H_r^{N_y}$ for various values of r and N_y

r	$N_y = 2$	$N_y = 3$	$N_y = 4$	$N_y = 5$
1	3.053	4.012	5.004	6.002
2	4.004	4.136	4.997	5.950
3	0.3511	6.076	6.014	7.119
4	1.982	0.618	7.221	6.259

TABLE-3

Nondimensional frequency parameter u for a 3x3 square Isotropic grid, S.S. on the edges.

$E_b = 30 \times 10^6$ psi, $\gamma_b = .284$, $A_b = 3$ in.^{2}, $I_b = 0.25$ in.^{4}, a = 50 in.

Mode	Ellington[6]	Chang[3]	Thein Wah[23]	Orthotropic[7] plate theory	Cheng[4]	Finite element method	Proposed method
B_{11}	0.7853	0.7853	0.7854	0.7854	0.7854	0.7854	0.7854
B_{12}	1.336	1.3411	1.339	1.3406	1.3393	1.3400	1.3400
B_{22}	1.565	1.5679	1.571	1.5708	1.5708	1.5708	1.5708
B_{13}	1.924	1.9874	1.963	1.9878	1.9629	1.9823	1.9874
B_{23}	2.061	2.0767	2.057	2.0734	2.0569	-	2.0726
B_{33}	2.281	2.3192	2.356	2.3562	2.3562	-	2.3562

TABLE 4-a

Frequencies (cps) of a 2x3 orthotropic rectangular grid S.S. at the edges.

$E_x = E_y = 30\times10^6$ psi , $\nu_x = \nu_y = .284$, $A_x = 3$ in^2, $A_y = 1.875$ in^2, $I_x = 0.25$ in^4, $I_y = 0.088$ in^4, a=50 inch, a/b = 1.332

Mode No.	Ellington[6]	Finite Element method	Proposed method
B_{11}	53.7	41.78	41.7
B_{21}	84.8	119.02	117.8
B_{12}	153.0	123.31	124.4
B_{22}	198.2	167.85	167.0
B_{31}	209.0	258.62	260.0

TABLE 4-b

Frequencies (cps) of 3 x 4 orthotropic rectangular grid, S.S. at the edges.

a/b = 1.25

Mode No.	Ellington[6]	Finite Element method	Proposed method
B_{11}	47.5	39.19	39.1
B_{21}	81.25	110.15	110.1
B_{12}	154.2	118.13	117.5
B_{22}	178.9	157.11	156.0
B_{31}	190.02	235.43	241.5

TABLE 5-a

Frequencies (cps) of a2x3 orthotropic rectangular grid, S.S. at the edges.

$E_x = 30 \times 10^6$ psi, $E_y = 12 \times 10^6$ psi, $\nu_x = .24$, $\nu_y = .1$,
$A_x = A_y = 3$ in.^{2}, $I_x = I_y = 0.25$ in.^{4} a = 50 inch, a/b = 1.332

Mode No.	Ellington[6]	Finite Element method	Proposed method
B_{11}	56.3	47.43	47.3
B_{21}	88.9	131.24	130.15
B_{12}	168.1	142.76	142.6
B_{22}	210.5	189.53	189.2
B_{31}	230.5	286.87	284.2

TABLE 5-b

Frequencies (cps) of a 3 x 4 orthotropic rectangular grid, S.S. at the edges.

a/b = 1.25

Mode No.	Ellington[6]	Finite Element method	Proposed method
B_{11}	52.5	44.31	44.2
B_{21}	86.2	128.07	128.1
B_{12}	153.5	129.97	129.5
B_{22}	192.3	177.44	176.8
B_{31}	225.5	286.18	281.0

TABLE 6-a

Frequencies (cps) of 2x2 orthotropic square grid, S.S. at the edges.

$E_x = E_y = 30 \times 10^6$ psi, $\nu_x = \nu_y = 0.284$, $A_x = 3$ in^2, $A_y = 1.875$ in^2, $I_x = 0.25$ in^4, $I_y = 0.088$ in^4, $a = 50$ inch

Mode No.	Ellington[6]	Finite Element method	Proposed method
B_{11}	33.5	33.45	33.40
B_{12}	72.2	73.50	73.90
B_{21}	112.8	116.04	116.00
B_{22}	129.4	135.01	133.20

TABLE 6-b

Frequencies (cps) of 3 x 3 orthotropic square grid, S.S. at the edges.

Mode No.	Ellington[6]	Finite Element method	Proposed method
B_{11}	33.0	33.43	33.40
B_{12}	73.6	73.92	73.90
B_{21}	115.2	114.65	116.00
B_{22}	133.0	134.17	133.20
B_{13}	147.2	152.41	155.00

TABLE 6-c

Frequencies (cps) of 4 x 4 orthotropic square grid,

S.S. at the edges.

Mode No.	Ellington[6]	Finite Element method	Proposed method
B_{11}	33.25	33.42	33.40
B_{12}	73.7	73.97	73.90
B_{21}	115.00	116.28	116.00
B_{22}	133.00	133.91	133.20
B_{13}	155.00	155.16	155.00

TABLE 7-a

Frequencies (cps) of a 2 x 2 square orthotropic grid,

S.S. on the edges.

$E_x = 30x10^6$psi, $E_y = 12x10^6$psi, $\nu_x = .284$, $\nu_y = .1$,
$A_x = A_y = 3$ in^2, $I_x = I_y = 0.25$ in^4 , $a = 50$ inch.

Mode No.	Ellington[6]	Finite Element method	Proposed method
B_{11}	37.00	37.32	37.25
B_{12}	84.2	84.58	85.7
B_{21}	125.0	123.13	127.2
B_{22}	144.5	150.90	148.2

TABLE 7-b

Frequencies (cps) of a 3 x 3 square orthotropic grid,

S.S. on the edges.

Mode No.	Ellington[6]	Finite Element method	Proposed method
B_{11}	37.2	37.30	37.25
B_{12}	85.0	85.51	85.7
B_{21}	126.5	127.31	127.2
B_{22}	147.2	149.74	148.2
B_{13}	172.5	175.64	182.0

TABLE 7-c

Frequencies (cps) of a 4 x 4 square orthotropic grid,

S.S. on the edges.

Mode No.	Ellington[6]	Finite Element method	Proposed method
B_{11}	37.23	37.29	37.25
B_{12}	85.52	85.66	85.7
B_{21}	127.0	127.73	127.2
B_{22}	147.9	149.40	148.2
B_{13}	178.2	180.40	182.0

TABLE 8-a

Frequencies (cps) of a 2 x 3 isotropic rectangular grid,

S.S. on the edges.

$E_b = 30x10^6$ psi $A_b = 3$ in^2 $I_b = 0.25$ in^4, $\nu_b = .284$, a = 50in.

a/b = 1.332

Mode No.	Ellington[6]	Finite Element method	Proposed method
B_{11}	52.6	52.87	52.8
B_{21}	112.5	112.47	113.2
B_{12}	179.9	178.36	196.2
B_{22}	204.9	224.51	211.5
B_{31}	223.2	-	237.0

TABLE 8-b

Frequencies (cps) of a 3 x 4 isotropic rectangular grid,

S.S. on the edges

a = 50 in   a/b = 1.25

Mode No.	Ellington[6]	Finite Element method	Proposed method
B_{11}	47.7	48.08	48.05
B_{21}	110.9	112.16	111.3
B_{12}	163.0	-	164.2
B_{22}	189.5	-	192.5
B_{31}	228.5	-	237.0

TABLE 9-a

Frequencies (cps) of a 2x2 isotropic square grid,

S.S. at the edges.

$E_b = 30x10^6$ psi, $A_b = 3$ in^2, $I_b = 0.25$ in^4, $\nu_b = 0.284$,

a = 50 inch.

Mode No.	Ellington[6]	Finite Element method	Proposed method
B_{11}	36.6	36.67	36.64
B_{12}	103.6	106.34	106.8
B_{21}	103.6	106.34	106.8
B_{22}	142.0	148.31	146.6

TABLE 9-b

Frequencies (cps) of a 3x3 isotropic square grid,

S.S. at the edges.

Mode No.	Ellington[6]	Finite Element method	Proposed method
B_{11}	36.59	36.65	36.64
B_{12}	106.00	106.79	106.8
B_{21}	106.00	106.87	106.8
B_{22}	145.10	147.18	146.6
B_{13}	241.00	231.08	234.6

TABLE 9-c

Frequencies (cps) of a 4x4 isotropic square grid,

S.S. at the edges.

Mode No.	Ellington[6]	Finite Element method	Proposed method
B_{11}	36.59	36.65	36.64
B_{12}	106.4	106.71	106.8
B_{21}	106.4	106.83	106.8
B_{22}	146.2	144.67	146.6
B_{13}	232.2	233.96	234.6

TABLE 10-a

Fundamental frequencies (cps) of rectangular orthotropic

grids clamped on the edges.

$E_x = E_y = 30 \times 10^6$ psi, $A_x = 3$ in^2, $A_y = 1.875$ in^2,
$I_x = 0.25$ in^4, $I_y = 0.088$ in^4, $\nu_x = \nu_y = 0.284$,
$a = 50$ in

Order of the grid	a/b ratio	Finite Element method	Proposed method
2 x 3	1.332	94.76	93.4
3 x 4	1.25	88.89	88.2
4 x 5	1.2	85.72	85.5

TABLE 10-b

Fundamental frequencies (cps) of square orthotropic grid clamped on the edges

Order of the grid	Finite Element method	Proposed method
2 x 2	76.06	75.5
3 x 3	75.86	75.5
4 x 4	75.80	75.5
5 x 5	75.73	75.5

TABLE 11-a

Frequencies (cps) of a 2 x 3 isotropic rectangular grid clamped on the edges

$E_b = 30 \times 10^6$ psi , $A_b = 3$ in^2, $I_b = 0.25$ in^4, $\gamma_b = .284$,

$a = 50$ in, $a/b = 1.332$

Mode No.	Finite Element method	Proposed method
B_{11}	119.79	117.2
B_{21}	193.05	189.0
B_{12}	270.02	259.0
B_{22}	309.02	303.0

TABLE 11-b

Frequencies (cps) of a 3 x 4 isotropic rectangular grid clamped on the edges

$a/b = 1.25$

Mode No.	Finite Element method	Proposed method
B_{11}	109.02	107.8
B_{21}	186.24	182.0
B_{12}	256.77	250.0
B_{22}	300.66	282.0

TABLE 11-c

Frequencies (cps) of 4 x 5 isotropic rectangular grid clamped on the edges

$a/b = 1.2$

Mode No.	Finite Element method	Proposed method
B_{11}	103.01	100.7
B_{21}	182.74	180.5
B_{12}	239.85	232.0
B_{22}	284.37	256.2

TABLE 12-a

Frequencies (cps) of a 2 x 2 square isotropic grid clamped on the edges

$E_b = 30 \times 10^6$ psi, $A_b = 3$ in^2, $I_b = 0.25$ in^4, $\gamma_b = .284$, $a = 50$ in.

Mode No.	Finite Element method	Proposed method
B_{11}	83.41	82.8
B_{12}	162.04	160.8
B_{21}	162.04	160.8
B_{22}	233.55	225.2

TABLE 12-b

Frequencies (cps) of a 3 x 3 square isotropic grid clamped on the edges

Mode No.	Finite Element method	Proposed method
B_{11}	83.18	82.8
B_{12}	170.89	170.0
B_{21}	170.89	170.0
B_{22}	231.10	225.2
B_{13}	292.10	291.1

TABLE 12-c

Frequencies (cps) of a 4 x 4 square isotropic grid clamped on the edges.

Mode No.	Finite Element method	Proposed method
B_{11}	83.11	82.8
B_{12}	171.98	170.2
B_{21}	171.98	170.2
B_{22}	229.90	225.2
B_{13}	316.04	302.0

TABLE 12-d

Frequencies (cps) of a 5 x 5 square isotropic grid clamped on the edges

Mode No.	Finite Element method	Proposed method
B_{11}	83.09	82.8
B_{12}	172.18	171.1
B_{21}	172.18	171.1
B_{22}	229.43	226.1
B_{13}	320.89	319.0

TABLE 13-a

First mode shape of 2 x 2 orthotropic grid, S.S. on the edges, with same dimension as given in Table 6-A

Finite	1.000	1.000	1.004	13.801	0.500	0.502
	13.801	0.500	-0.502	-1.004	1.004	13.801
Element	-0.500	0.502	13.801	-0.500	-0.502	-1.004
	-1.000	-1.000				
Method						
Proposed	1.000	1.000	1.000	13.783	0.500	0.500
	13.783	0.500	-0.500	-1.000	1.000	13.783
Method	-0.500	0.500	13.783	-0.500	-0.500	-1.000
	-1.000	-1.000				

TABLE 13-b

First mode shape of 3 x 3 orthrotropic grid, S.S. on the edges

Finite	1.000	1.414	1.000	1.001	11.258	0.707
	0.708	15.921	1.000	0.000	11.258	0.707
Element	-0.708	-1.001	1.416	15.921	0.000	1.001
	22.516	0.000	0.000	15.921	0.000	-1.001
Method	-1.416	1.001	11.258	-0.707	0.708	15.921
	-1.000	0.000	11.258	-0.707	-0.708	-1.001
	-1.000	-1.414	-1.000			
Proposed	1.000	1.414	1.000	1.000	11.254	0.707
	0.707	15.916	1.000	0.000	11.254	0.707
Method	-0.707	-1.000	1.414	15.916	0.000	1.000
	22.508	0.000	0.000	15.916	0.000	-1.000
	-1.414	1.000	11.254	-0.707	0.707	15.916
	-1.000	0.000	11.254	-0.707	-0.707	-1.000
	-1.000	-1.414	-1.000			

TABLE 13-c

Second mode shape of a 2 x 2 orthotropic grid, S.S. at the edges .

Finite	1.000	-1.000	1.754	13.472	0.500	-0.877
	-13.471	-0.500	-0.877	1.754	1.754	13.472
Element	-0.500	-0.877	-13.472	0.500	-0.877	1.754
	-1.000	1.000				
Method						
Proposed	1.000	-1.000	2.000	13.750	0.500	-0.882
	-13.750	-0.500	-0.882	2.000	2.000	13.750
Method	-0.500	-0.882	-13.750	0.500	-0.882	2.000
	-1.000	1.000				

TABLE 13-d

Second mode shape of a 3 x 3 orthotropic grid, S.S. on the edges.

Finite	1.000	0.000	-1.000	1.269	11.175	0.707
	0.000	0.000	0.000	-1.369	-11.175	-0.707
Element	0.000	1.396	1.936	15.305	0.000	0.000
	0.000	0.000	-1.936	-15.805	0.000	0.000
Method	1.936	1.369	11.175	-0.707	0.000	0.000
	0.000	-1.369	-11.175	0.707	0.000	1.369
	-1.000	0.000	1.000			
Proposed	1.000	0.000	-1.000	1.423	11.231	0.723
	0.000	0.000	0.000	-1.423	-11.231	-0.723
Method	0.000	1.423	2.000	15.850	0.000	0.000
	0.000	0.000	-2.000	-15.850	0.000	0.000
	2.000	1.423	11.231	-0.723	0.000	0.000
	0.000	-1.423	-11.231	0.723	0.000	1.423
	-1.000	0.000	1.000			

APPENDIX -C

COMPUTER

PROGRAMME

```
C         DYNAMIC ANALYSIS OF ORTHOTROPIC
C         RECTANGULAR GRID SIMPLY SUPPORTED ON THE BOUNDARIES.
C         THIS PROGRAM GENERATES THE MASS AND STIFFNESS
C         MATRICES OF THE GRID AND CALCULATES EIGEN VALUES
C         AND EIGENVECTORS OF THE GRID.
C         S REPRESENTS STIFFNESS MATRIX.
C         G REPRESENTS THE MASS MATRIX.
C         AR=AREA OF CROSSECTION.W=WEIGHT DENSITY.
C         BR=BREDTH,H=HEIGHT OF THE CROSSECTION.
C         E=ELASTICITY CONSTANT, DR=MOMANT OF INERTIA,
C         X=LENGT., R=MASS DENSITY.
          COMMON/B1/S(50,50)
          COMMON/B2/G(50,50)
          COMMON/B3/ND
          COMMON/B4/C(50,50)
          DATA E,BR,H, X1, W/30.0E+06, 3.,1.,50.0,0.284/
          DATA E1,BR1,H1, W1/12, 0E+06,3.,1.,.1/
          DR=BR*H**3/12.
          DR1=BR1*H1**3/12.
          AR=BR*H
          AR1=BR1*H1
          R=W/386.4
          R1=W1/386.4
C         NY REPRESENTS THE NO. OF BEAMS IN X-DIRECTION
C         AND NX REPRESENTS NO. OF BEAMS IN Y-DIRECTION
          DO1000NY=2,4,1
      45  FORMAT(10X,*C1=*,E15.5,*C2=*E15.5,*C3=*, E15.5,
          *C4=*,B15.5)
      46  FORMAT(1X,*ORDER OF THE GRID IS=*,I2,*BY*,I2)
          NX=NY+1
          MS=NX+1
          X=X1/FLOAT(MS)
          C1=E*DR/X**3
          C2=R*AR*X/420.
          PRINT45,C1,C2,C3,C4
C         NA IS THE ORDER OF THE MASS AND STIFFNESS MATRICES
          NA=3*NY**2+7*NY+2
          DO1I=1,NA
          DO1J=1,NA
          G(I,J)=0.
       1  S(I,J)=0
          DO2I=1,NY
          S(I,I)=C1*4.*X**2
          G(I,J)=C2**4.*X**2
       2  CONTINUE
          NC=NA-NY
          DO3I=1,NY
          ND=NC+I
          S(ND,ND)=C1*4.*X**2
          G(ND,ND)=C2*4.*X**2
```

```
 3    CONTINUE
      DO4I=1,NY
      J=3*I-2
      NF=NX+J
      S(I,NF)=-C1*6.*X
      S(I,NF+1)=C1*2.*X**2
      G(I,NF)=C2*13.*X
      G(I,NF+1)=-C2*3.*X**2
 4    CONTINUE
      DO5J=1,NY
      I=3*J-2
      NI=3*NY**2+3*NY+1+I
      NJ=(5*NY**2+17*NY)/2-1+J
      S(NI,NJ)=C1*6.*X
      S(NI+1,NJ)=C1*2.*X**2
      G(NI,NJ)=-C2*13.*X
      G(NI+1,NJ)=-C2*3.*X**2
 5    CONTINUE
      NL=4*NY+3
      NI=3*NY-2
      DO7K=1,NI,3
      NM=NX+K
      NN=NL+K
      S(NM,NN)=-C1*12.
      S(NM,NN+1)=C1*6.*X
      S(NM+1,NN)=-C1*6.*X
      S(NM+1,NN+1)=C1*2.*X**2
      G(NM,NN)=C2*54.
      G(NM,NN+1)=-C2*13.*X
      G(NM+1,NN)=C2*13.*X
      G(NM+1,NN+1)=-C2*3.*X**2
 7    CONTINUE
      NP=3*NY+2
      IL=NY-1
      DO6L=1,IL
      DO6K=1,NI,3
      DO6I=1,2
      DO6J=1,2
      IM=NX+I
      IN=NL+J
      IQ=IM+L*NP
      IR=IN+L*NP
      IM=IM+K-1
      IN=IN+K-1
      NQ=NQ+K-1
      NR=NR+K-1
      S(NQ,NR)=S(IM,IN)
      G(NQ,NR)=G(IM,IN)
```

```
 6    CONTINUE
      S(NX,NX)=C3*4.*X**2
      S(NX,NX+1)=-C3*6.*X
      S(NX,NX+3)=C3*2.*X**2
      G(NX,NX)=C4*4.*X**2
      G(NX,NX+1)=C4*13.*X
      G(NX,NX+3)=-C4*3.*X**2
      NS=3*NY-5
      DO8K=1,NS,3
      NT=NX+K
      S(NT,NT)=24.*C1+24.*C3
      S(NT,NT+3)=-C3*12.
      S(NT,NT+5)=C3*6.*X
      S(NT+1,NT+1)=C1*8.*X**2
      S(NT+2,NT+2)=C3*8.*X**2
      S(NT+2,NT+3)=-C3*6.*X
      S(NT+2,NT+5)=C3*2.*X**2
      S(NT+3,NT+3)=24.*C1+24.*C3
      S(NT+4,NT+4)=C1*8.*X**2
      S(NT+5,NT+5)=C3*8.*X**2
      G(NT,NT)=312.*C2+312.*C4
      G(NT,NT+3)=C4*54.
      G(NT,NT+5)=-C4*13.*X
      G(NT+1,NT+1)=C2*8.*X**2
      G(NT+2,NT+2)=C4*8.*X**2
      G(NT+2,NT+3)=C4*13.*X
      G(NT+2,NT+5)=-C4*3.*X**2
      G(NT+3,NT+3)=312.*C2+312.*C4
      G(NT+4,NT+4)=C2*8.*X**2
      G(NT+5,NT+5)=C4*8.*X**2
 8    CONTINUE
      NU=4*NY+2
      S(NU,NU)=C3*4.*X**2
      S(NU-1, NU)=C3*2.*X**2
      S(NU-3,NU)=C3*6.*X
      G(NU,NU)=C4*4.*X**2
      G(NU-1,NU)=-C4*3.*X**2
      G(NU-3,NU)=-C4*13.*X
      DO9K=1,NY
      DO9I=1,NP
      DO9J=1,NP
      NV=I+K*NP+NY
      NW=J+K*NP+NY
      MA=NY+I
      MB=NY+J
      S(NV,NW)=S(MA,MB)
      G(NV,NW)=G(MA,MB)
```

```
    9     CONTINUE
          DO10K=2,NA
          MC=K-1
          DO10J=1,MC
          S(K,J)=S(J,K)
          G(K,J)=G(J,K)
   10     CONTINUE
          PRINT46,NY,NX
          CALL BINV
          CALL MATML
          CALL EIGNV
 1000     CONTINUE
          STOP
          END
£IBFTC   SUE1
C         THIS SUBROUTINE CALCULATE THE INVERSE OF AMATRIX
          SUBROUTINE  BINV
          COMMON/B1/A(50,50)
          COMMON/B3/N
          DIMENSION  INDEX(50,3)
          EQUIVALENCE (IROW,JROW),(ICOLUM,JCOLUM),(AMAX,T,
          SWAP)
   15     DO 20 J=1,N
   20     INDEX(J,3)=0
   30     DO 550 I=1,N
C         SEARCH FOR PIVOT ELEMENT
   40     AMAX=0.0
   45     DO105J=1,N
          IF(INDEX(J,N)-1)60,105,60
   60     DO 100K=1,N
          IF (INDEX(K,3)-1)80,100,80
   80     IF (AMAX-AVS(A(J,K)))85,100,100
   85     IROW=J
   90     ICOLUM=K
          AMAX=ABS(A(J,K))
  100     CONTINUE
  105     CONTINUE
          INDEX(ICOLUM,3)=INDEX(ICOLUM,3)+1
  260     INDEX(I,1)=IROW
  270     INDEX(I,2)=ICOLUM
C         INTERCHANGING ROWS TO PUT PIVOT ELEMENTS
C         ON DIOGONAL
  130     IF(IROW-ICOLUM)150,310,150
  150     DO 200 L=1,N
  160     SWAP=A(IROW,L)
  170     A(IROW,L)=A(ICOLUM,L)
  200     A(ICOLUM,L)=SWAP
C         DIVIDE PIVOT ROW BY PIVOT ELEMENT
```

```
 310     PIVOT=A(ICOLUM,ICOLUM)
 330     A(ICOLUM, ICOLUM)=1.0
 340     DO350L=1,N
 350     A(ICOLUM,L)=A(ICOLUM,L)/PIVOT
C        REDUCE NON-PIVOT ROWS
 380     DO550L1=1,N
 390     IF (L1-ICOLUM)400,550,400
 400     T=A(L1,ICOLUM)
 420     A(L1,ICOLUM)=0.0
 430     DO450L=1,N
 450     A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
 550     CONTINUE
C        INTERCHANGING COLUMNS
 600     DO710I=1,N
 610     L=N+1-I
 620     IF(INDEX(L,1)-INDEX(L,2))630,710,630
 630     JROW=INDEX(L,1)
 640     JCOLUM=INDEX(L,2)
 650     DO705K=1,N
 660     SWAP=A(K,JROW)
 670     A(K,JROW)=A(K,JCOLUM)
 700     A(K,JCOLUM)=SWAP
 705     CONTINUE
 710     CONTINUE
 740     RETURN
         END
C        THIS SUBROUTINE IS TO CALCULATE THE
C        MULTIPLICATION OF TWO MATRICES
         SUBROUTINE MATML
         COMMON/B1/S(50,50)
         COMMON/B2/G(50,50)
         COMMON/B3/NA
         COMMON/B4/D(50,50)
         DO1I=1,NA
         DO1J=1,NA
         D(I,J)=0.
         DO1K=1,NA
         D(I,J)=D(I,J)+S(I,K)*G(K,J)
   1     CONTINUE
         RETURN
         END
£IBFTC  EIGNV
C        THIS SUBROUTINE CALCULATES EIGEN-VALUES AND
C        EIGEN-VECTORS
         SUBROUTINE EIGNV
         COMMON/B3/M
         COMMON/B4/C(50,50)
         DIMENSION EIGN(9),Y(50),X(50),Y1(50) Z(50)
         DIMENSION X2(50),X1(50),D(1,50)
   4     FORMAT(I2,*TM. EINENVALUE=*,E16.8,10X,*FRE-
        1JENCY=*,F10.2,10X,I5)
   5     FORMAT(I2,*TH.EIGENVECTOR*)
   6     FORMAT (8F10.5)
```

```
      EQUIVALENCE(Y1,Z)
      E=0.00001
      DO110IV=1,5
      JK=IV
      ITER=0
      IF(M.EQ.1)  Y(1)=1.0
      IF(IV-M)116,117,116
116   N=M-IV+1
      N1=N-1
      DO100 I=1,N
100   Y1(I)=1.0
99    KC=0
C     AND EIGENVECTORS ARE CARRIED OUT
      DO 101 I=1,N
      Y(I)=0.0
      DO  101  J=1,N
101   Y(I)=Y(I)+C(I,J)*Y1(J)
      R=Y(1)
      DO  90  I=1,N
      Y(I)=Y(I)/R
      IF(ABS(Y(I)/Y1(I)-1.0)-E)   90,90,102
102      KC=1
90    CONTINUE
      ITER=ITER+1
      IF(ITER.GE.300) GO TO 103
      IF(KC)  103, 103, 104
104   DO115  I=1,N
115   Y1(I)=Y(I)
      GO  TO  99
C     UPTO THE STATEMENT 113, THE SIZE OF THE C
C     MATRIX IS REDUCED BY ONE
103   EIGN(IV)=R
      DO  109  I=1,N1
109   X(I)=-Y(I+1)
      DO 121  J=2,N
121   Z(J)=C(1,J)
      IF(IV.GT.1) GO TO 8
      DO7I=1,N
7     X2(I)=Y(I)
      DO1J=1,M
1     D(1,J)=C(1,J)
8     CONTINUE
      DO  112 I=2,N
      DO  112 J=2,N
112   C(I-1,J-1)=C(I,J)+Z(J)*X(I-1)  .
      GO TO 118
117   EIGN(IV)=C(1,1)
```

```
 118    FREQ =1.0/(SQRT(EIGN(IV))*6.2832)
        PRINT 4,IV,EIGN(IV),FREQ,ITER
        IF(IV.EQ.1) GO TO 141
        IF(IV.GT.2) GO TO 110
        X1(1)=0.
        IC=M-1
        DO2I=1,IC
   2    X1(I+1)=Y(I)
        SUM=0.
        DO9I=1,M
   9    SUM=SUM+D(1,I)*X1(I)
        F=(EIGN(2)-EIGN(1))/SUM
        DO3I=1,M
   3    Y(I)=X2(I)+F*X1(I)
 141    PRINT 6,(Y(I),I=1,M)
 110    CONTINUE
        RETURN
        END
£ENTRY
```